1 Introduction

The ion loss and the confinement.

2 Experiments

Paper **Hauff Jenko 2008**: fast broadening of the current profile when the NBI is off-axis and no observed MHD activity; finding: transport levels depend on the fast particles orbit.

In $2D$ there is gyroaveraging : the circle of Larmor gyration, with radius ρ_g covers a small region of fluctuating electrostatic potential and it produces an average effect of it. The averaging of the electrostatic fluctuations reduces the electric drift velocity. This reduction, in turn, acts to decrease the transport rate. Then: *for small Kubo numbers, the increase of the radius of gyration leads to decrease of the rate of transport.*

3 Ion orbit loss rate in tokamak (Shaing [?])

To explain the H-mode in tokamaks. The loss of ions is localized in the high energy part of the distribution function, since here the ions are less collisional. Being less collisional, they have rather clear banana trajetories and in their motion they can hit the limiter.

But the expulsion of a "hot" ion from the plasma is simultaneously compensated (electrically) by the entrance of an ion from the exterior of the plasma toward the interior. This influx is driven by the ion viscosity which is essentially determined by a non-zero rotation of the plasma. It concerns the lower temperature ions, *i.e.* the ions of the Pfirsch-Schluter-plateau, more collisional. This again rises the problem of Stix : it is necessary to have the poloidal rotation + collisions in order to get a radial electric current. This current will participate in the radial component of the $\nabla \times \mathbf{B}$ equation, to compensate (together with the polarization electric current) the outflux of directly lost hot ions. Also in Roenbluth it is the current of compensation of the lost very-hot-ions at NBI.

This problem of balance of ion radial currents can be re-stated: the loss of ions is the primary process; but the plasma establishes an electric radial field andthe corresponding rotation in order to obtain, through the ion-viscosity, a radial electric current of ions which balances the outgoing ion flux.

- the *outgoing* ion orbit loss flux (in **banana regime**) , is balanced by
- incoming *viscosity driven* ion flux (plateau-Pfirsch-Schluter)

to mentain ambipolarity at the steady state.

Nota. It must be understood that the *disappearence of an ion (* whose banana orbit hits the wall or the limitator) from the plasma **should not be seen as a current directed to the wall**. One expects that the ion which will replace the lost ion, will come from the plasma border toward the centre. So, there is a current of response, the so-called *incoming current*.

The effect of viscosity is separated in two in order to emphasize the two different effects:

1. "viscosity-driven flux" is the flux driven by the viscosity contributed by the particles in the regime **plateau-Pfirsch-Schluter**. Actually is the flux of ions of replacement.

2. "ion orbit loss flux" is the flux driven by the viscosity of the particles in the banana regime. (This is because the loss of ions is due to the difference in the radial drifts of the electrons and the ions, when the drifts are not too much perturbed by the collisions , *i.e.* in the banana regime). But it is not clear how the viscosity is involved in the **direct loss of banana ions to the limiter**.

When the two fluxes (outcoming and incoming) are integrated over the velocity space, they *approximately cancel* each other. The **net** ion flux cancels to order $\sqrt{m_e/m_i}$ *when E_r is determined proprely from the momentum balance equation.*

NOTE. In the paper Shaing insists on the difference between the *ion loss current* and *the plasma current density*. The later is the current formed by ions which replaces the ions lost by the intersection of their banana with the wall.

The torque associated to the ion orbit loss flux is counterbalanced by the torque associated with the viscosity. At steady state there is no net radial current across flux surfaces and there is no net torque applied on the plasma.

Various currents which constitutes the radial plasma current:

- the **ion orbit loss current** $e\Gamma_{orbit}$
- the **viscosity-driven** current;
- the **polarization** current $\sim \frac{\partial E_r}{\partial t}$ up to saturation. See **Honda**.

At *steady state*:

- the radial current density j_r which is proportional with $\partial E_r/\partial t$ **and** the polarization current vanish
- the ion orbit loss current $e\Gamma_{orbit}$ is balanced by the viscosity-driven flux

The equation used by Shaing

$$\left(v_{\parallel}\widehat{\mathbf{n}}+\mathbf{V}_E\right)\cdot\boldsymbol{\nabla}\theta\frac{\partial f}{\partial\theta}+\mathbf{v}_d\cdot\boldsymbol{\nabla}\psi\frac{\partial f}{\partial\psi}=C\left(f\right)$$

where $\mathbf{V}_E$ is the electric velocity. The equation is simply $\left(v_{\parallel}\widehat{\mathbf{n}}+\mathbf{v}_d+\mathbf{V}_E\right)\cdot\boldsymbol{\nabla}f=C\left(f\right)$:

- *without explicit time derivative* (since the process here is not dynamic, as for example when we study the decay of plasma rotation by the torque generated from the radial electric current of ions (compared to electrons) in the presence of **rotation** and **collisions**, see Stix;
- also, without *energetic term* in the drift kinetic equation; this should be accounted for in other cases, as for example the **transit time magnetic pumping**, where the viscosity is noncollisional. This appears in [**?**] where the equation is $(u\widehat{\mathbf{n}} + \mathbf{v}_d + \mathbf{V}) \cdot \boldsymbol{\nabla} f + \dot{w}\partial f/\partial w = 0$; and the equation is so because there is a strong plasma rotation composed of : *diamagnetic*, *electric* and *parallel*.

The equation for the distribution function is a static balance of convective terms and collisions.

The $\mathbf{v} \cdot \boldsymbol{\nabla} f$ part of the equation (left hand side) The first part in the equation is of this form because the drift kinetic distribution function f is function of only the **poloidal** θ coordinate and **radial** r coordinate. The variables are (the ion charge is e, *i.e.* $e = |e|$):

$$\begin{aligned} & \psi, \theta \\ \epsilon \quad = \quad & \frac{v^2}{2} - \frac{|e|}{m_i}\Phi \\ & \mu \end{aligned}$$

The effects of orbit squeezing S can be taken into account by employing a new coordinate ψ_* istead of ψ.

$$\psi_* = \psi - \frac{I}{S\Omega}\left(v_\parallel + \frac{I}{\Omega}\frac{B^2}{B_0^2}\frac{e}{m_i}\frac{\partial \Phi}{\partial \psi}\right)$$

where the **squeezing factor** is

$$S = 1 + \left(\frac{I}{\Omega_0}\right)^2 \frac{e}{m_i}\frac{\partial^2 \Phi}{\partial \psi^2}$$

NOTE. Not connected bu for comparison, the variable ψ_* representing the *drift surface* is given in [**?**] as

$$\psi_* \equiv \psi - \frac{v_\parallel}{\Omega} = \text{const}$$

(we could say $\psi_* = \psi - \rho_\parallel$ but take the *variable* poloidal Larmor radius, with the parallel velocity variable up to zero and change of direction, and with B also variable). Here the function ψ is

$$\psi = \frac{\Psi}{RB_\varphi}$$

where Ψ is the poloidal magnetic flux function. A reasonable approximation for the magnetic field variation in space

$$RB_\varphi = \text{const}$$

If we assume *a constant current density* we obtain

$$\psi = \frac{1}{qR}\frac{r^2}{2}$$

END of the NOTE

The quantity Ω_0 is Ω calculated at the magnetic axis. The shear of the electric field Φ'' is considered constant over the width of the banana orbit.

It can be shown that

$$\omega = \left(v_\parallel \widehat{\mathbf{n}} + \mathbf{V}_E\right) \cdot \boldsymbol{\nabla}\theta \simeq -\frac{I}{\Omega}\widehat{\mathbf{n}} \cdot \boldsymbol{\nabla}\theta \frac{\partial\psi_*/\partial\psi}{\partial\psi_*/\partial\epsilon}$$

and

$$\mathbf{v}_d \cdot \boldsymbol{\nabla}\psi = \frac{I}{\Omega}\widehat{\mathbf{n}} \cdot \boldsymbol{\nabla}\theta \frac{\partial\psi_*/\partial\theta}{\partial\psi_*/\partial\epsilon}$$

where we have

$$\begin{aligned} \widehat{\mathbf{n}} \cdot \boldsymbol{\nabla}\theta &= \nabla_\parallel \theta = \frac{\Delta\theta}{\Delta l_\parallel} = \frac{1}{r}\frac{r\Delta\theta}{\Delta l_\parallel} = \frac{1}{r}\frac{B_\theta}{B} \\ &\approx \frac{1}{qR} \end{aligned}$$

The meaning of the notation is

$$I = R^2 \mathbf{B} \cdot \boldsymbol{\nabla}\varphi = RB_\varphi$$

Note

similar equations used by **Rosenbluth Hazeltine Hinton 1972**

$$\begin{aligned} \mathbf{v}_D \cdot \boldsymbol{\nabla}\left(v_\varphi h\right) &= 0 \quad \text{implies} \\ v_{D\theta} &= -\frac{\frac{\partial(v_\varphi h)}{\partial r}}{\frac{\partial(v_\varphi h)}{r\partial\theta}} \times v_{Dr} \end{aligned}$$

End.

The drift associated with the poloidal field variation $\partial B/\partial\theta$ have been neglected. Taking this relation into the drift-kinetic equation we get

$$\left(v_\parallel \widehat{\mathbf{n}} + \mathbf{V}_E\right) \cdot \boldsymbol{\nabla}\theta \frac{\partial f}{\partial\theta} + \mathbf{v}_d \cdot \boldsymbol{\nabla}\psi \frac{\partial f}{\partial\psi} \simeq \omega \left.\frac{\partial f}{\partial\theta}\right|_{\psi_*, E, \mu}$$

We note that ω is a frequency which has the role of a coordinate which combines the parallel velocity and electric $\mathbf{E} \times \mathbf{B}$ particle velocities **projected on the poloidal direction**, divided at the local small radius r. **Note** We must check the flux variable ψ_* with the "drift surface" variable, as introduced in the review of Hazeltine and Hinton.

The collision operator Consider only the **pitch-angle scattering** operator

$$C(f) = \frac{\nu_D}{2} \frac{\partial}{\partial \xi} \left(1 - \xi^2\right) \frac{\partial f}{\partial \xi}$$

where ν_D is the deflection collision frequency and

$$\xi = \frac{v_\parallel}{v}$$

is the **pitch angle**.

Change of variables

$$(\psi, \xi) \rightarrow (\psi_*, \omega)$$

Keeping the highest order derivatives, one obtains

$$C(f) \simeq \frac{\nu_D}{2} \left[(v\widehat{\mathbf{n}} \cdot \boldsymbol{\nabla}\theta)^2 \frac{\partial^2 f}{\partial \omega^2} - 2 \frac{I v^2 \widehat{\mathbf{n}} \cdot \boldsymbol{\nabla}\theta}{\Omega S} \frac{\partial^2 f}{\partial \omega \partial \psi_*} + \left(\frac{I v}{\Omega S} \right)^2 \frac{\partial^2 f}{\partial \psi_*^2} \right]$$

Here it has been approximated $1 - \xi^2 \approx 1$. This is valid since **it is the barely circulating and the barely trapped particles which contribute to the ion orbit loss**, *i.e.* $v_\parallel \rightarrow 0$.

A new variable is introduced, $\widehat{\omega}$ which makes possible to connect these expressions which will finally give f with the *neoclassical distribution function for ions*:

$$\omega = \sigma \widehat{\omega} \; \widehat{\mathbf{n}} \cdot \boldsymbol{\nabla}\theta \; \left(1 - k \sin^2 \frac{\theta}{2}\right)^{1/2}$$

Then

- $k < 1$ corresponds to *poloidally circulating particles,* and
- $1 < k < \infty$ corresponds to the *poloidally trapped particles.*

Here the **direction** of the variable ω is $\sigma = \pm 1$.

Another change of variables is suggested by the form of the **drift flux function**. We take

$$\psi_* = \psi_0 - \frac{I}{\Omega S} \widetilde{\omega}_0$$

assuming

$$\varepsilon \ll 1 \;\text{ but }\; |S| \, \varepsilon < 1$$

then

$$\begin{aligned} \widehat{\omega}^2 &= \overline{\omega}_0^2 \frac{B_0^2}{B_x^2} \\ &\quad + 2SE \\ &\quad + 2 |S| \, \varepsilon \left[\mu B_0 + \left(\overline{\omega}_0 \frac{B_0}{B_x} - I \frac{1}{B_0} \frac{\partial \Phi}{\partial \psi} \right)^2 \right] \end{aligned}$$

where

$$E = \epsilon - \mu B_0 - \frac{e\Phi}{m_i} - \frac{1}{2}\left(\overline{\omega}_0 \frac{B_0}{B_x} - I\frac{1}{B_0}\frac{\partial \Phi}{\partial \psi}\right)^2$$

The notation $\widetilde{\omega}_0$ represents $\overline{\omega}_0 B/B_x$ where

$$\overline{\omega}_0 = v_{\parallel} + I\frac{1}{B_0}\frac{\partial \Phi}{\partial \psi}$$

evaluated at $\psi = \psi_0$ and $\theta = 0$ or π (depending on where the particles are trapped: $\theta = 0$ for inside of a tokamak, $\theta = \pi$ if the particle is trapped outside of the tokamak). B_x is the value of B evaluated at $\theta = 0$ or $\theta = \pi$.

3.0.1 Possibility of reprezentation of the effect of the ion orbit loss flux as a force in the balance equations

The effect of the ion orbit loss flux can be *modelled* by a force $\mathbf{\Sigma}$. The basis for this modellization is the fact that **every particle loss mechanism has a force which corresponds to it**. The example is the intrinsically nonambipolar flux of particles which is driven by the **viscous force**

$$\langle \mathbf{\Gamma}_{\pi} \cdot \boldsymbol{\nabla}\psi \rangle = -\left(\frac{\langle R^2 \rangle}{Ie}\right) \langle \mathbf{B} \cdot \boldsymbol{\nabla} \cdot \mathbf{\Pi} \rangle$$

(this is also in **Shaing Crume Houlberg**) .

Note in **Hirshman Sigmar review** (see *viscosity.tex*)

$$\overline{\Gamma}_a^{\psi} = -\left\langle \frac{1}{m_a \Omega_a} \left[\widehat{\mathbf{n}} \times \left(-\boldsymbol{\nabla} p_a - \boldsymbol{\nabla} \cdot \boldsymbol{\pi}_a + e_a n_a \boldsymbol{\nabla}\Phi\right)\right] \cdot \boldsymbol{\nabla}\psi \right\rangle$$

where

$$\overline{\Gamma}_a^{\psi} = \left\langle \int d^3v \ \left(\mathbf{v}_{Da} \cdot \boldsymbol{\nabla}\psi\right) \overline{f}_a \right\rangle$$

End.

[**Note** in **Hirshman Sigmar review**. See *viscosity.tex.*

$$\left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \left(n_a e_a \mathbf{u}_a \times \mathbf{B}\right)\right\rangle = \frac{\chi'}{2\pi} \left\langle R^2 \left|\boldsymbol{\nabla}\varphi\right|^2 \ n_a e_a u_a^{\psi} \right\rangle$$

and returning to the static force balance

$$0 = \left\langle n_a e_a \ R^2 \boldsymbol{\nabla}\varphi \cdot \mathbf{E} + R^2 \boldsymbol{\nabla}\varphi \cdot \mathbf{F}_{a1} \right\rangle + e_a \frac{\chi'}{2\pi} \Gamma_a^{\psi}$$

or

$$\Gamma_a^{\psi} = -\frac{1}{e_a}\frac{2\pi}{\chi'} \left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \left(\mathbf{F}_{a1} + e_a n_a \mathbf{E}\right)\right\rangle$$

End.]

[**Note** the static force balance is

$$0 = -\boldsymbol{\nabla} \cdot \boldsymbol{\Pi} + en\mathbf{v} \times \mathbf{B}$$

This is multiplied vectorially by $\mathbf{B}$,

$$\begin{aligned} 0 &= -\varepsilon^{ijk} B_j \ (\partial_p \ \Pi_{pk}) - e\Gamma^i B^2 \\ \text{assuming radial } \Gamma, \ \mathbf{v} \cdot \mathbf{B} &= 0 \end{aligned}$$

$$\Gamma^i = -\frac{1}{eB^2} \varepsilon^{ijk} B_j \ (\partial_p \ \Pi_{pk})$$

Now we extract from here the *radial* Γ_r

$$\Gamma_r = \frac{1}{|\boldsymbol{\nabla}\psi|} \boldsymbol{\nabla}\psi \cdot \boldsymbol{\Gamma} = \frac{1}{RB_\theta} (\partial_i \psi) \Gamma^i$$

and ask that the *surface average* $\langle \boldsymbol{\nabla}\psi \cdot \boldsymbol{\Gamma} \rangle$ is zero

$$\begin{aligned} \langle \boldsymbol{\nabla}\psi \cdot \boldsymbol{\Gamma} \rangle &= -\left\langle (\partial_i \psi) \frac{1}{eB^2} \varepsilon^{ijk} B_j \ (\partial_p \ \Pi_{pk}) \right\rangle \\ &= 0 \end{aligned}$$

End].

Similar to this example, *the flux that exists only in the presence of the* **ion orbit loss region** *in the phase space* is considered associated to a force $\boldsymbol{\Sigma}$. This method, of separating the ion orbit loss flux from the viscous flux is useful, since the viscosity can be calculated in the neoclassical approach but only if we ignore such processes ion orbit loss, which limits the integration on velocity.

Introducing this *force* $\boldsymbol{\Sigma}$, one can repeat the calculation of Hirschman to obtain

$$\begin{aligned} & \frac{1}{\langle B_p^2 \rangle} \left(\langle B^2 \rangle - \frac{I^2}{\langle R^2 \rangle} \right) \frac{\partial \langle \mathbf{V} \cdot \mathbf{B}_p \rangle}{\partial t} + \frac{I}{\langle R^2 \rangle nm_i} \langle \mathbf{J} \cdot \boldsymbol{\nabla}\psi \rangle \\ = \ & \frac{I}{\langle R^2 \rangle nm_i} \langle \mathbf{J}_{orb} \cdot \boldsymbol{\nabla}\psi \rangle - \frac{1}{nm_i} \langle \mathbf{B} \cdot \boldsymbol{\nabla} \cdot \boldsymbol{\Pi} \rangle + \left\langle \left(\frac{I^2}{\langle R^2 \rangle B^2} - 1 \right) \frac{\mathbf{B} \cdot \boldsymbol{\Sigma}}{nm_i} \right\rangle \end{aligned}$$

where the *radial current due to the orbit ion loss* is related to the force $\boldsymbol{\Sigma}$

$$\langle \mathbf{J}_{orb} \cdot \boldsymbol{\nabla}\psi \rangle = \left\langle \boldsymbol{\nabla}\psi \cdot \frac{\mathbf{B} \times \boldsymbol{\Sigma}}{B^2} \right\rangle$$

At steady state, the radial component of the $\boldsymbol{\nabla} \times \mathbf{B}$ equation which is

$$0 = \left(\mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \right)_r$$

will give

$$\begin{aligned} \mu_0 J_r &= 0 \\ \frac{\partial E_r}{\partial t} &= 0 \end{aligned}$$

However, the orbit loss curent $\mathbf{J}_{orb}$ is not zero. It is compensated by the viscosity driven current, which is seen from the equation at stationarity

$$\frac{I}{\langle R^2\rangle nm_i}\langle \mathbf{J}_{orb}\cdot\boldsymbol{\nabla}\psi\rangle + \left\langle \left(\frac{I^2}{\langle R^2\rangle B^2}-1\right)\frac{\mathbf{B}\cdot\boldsymbol{\Sigma}}{nm_i}\right\rangle = \frac{1}{nm_i}\langle \mathbf{B}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}\rangle$$

if we neglect the second term in the left hand side.

The meaning of this expression is that: *high energy ions are lost* and *the low energy ions respond by a current driven by their viscosity.*

4 Yushmanov. Electrostatic potential formation at the edge and rotation induced by suprathermal ion losses at the edge

The edge ion losses generate a radial electrostatic potential

This radial electric field modifies the orbits of the ions.

- the suprathermal ions are lost even if their orbit starts deep in the plasma. This is because the orbits are very large radially. We note that only the orbits of the ions that start on a particular magnetic surface and are directed such that they are *external* to the surface can be lost. Assume that $\mathbf{B}$ and $\mathbf{J}$ are parallel (both are "negative" in the plots of the JET discharges). Looking in the same direction as $\mathbf{B}$ we see the magnetic field lines turning down, as the right hand turns clockwise. For an ion whose the initial parallel velocity is directed like $\mathbf{B}$ the *drift* velocity acts vertical and places the banana entirely *inside* the magnetic surface. It cannot be lost. Those suprathermal ions that start with parallel velocity opposite (like $-\mathbf{B}$) will complete their large bananas *outside* the magnetic surface and can be lost. We conclude:
  - when the injection of the NBI ions is *counter-current* (which here also means *counter*-$\mathbf{B}$) there is a large population of suprathermal ions that are born with velocities directed like $-\mathbf{B}$.
  - these ions are possibly subject to loss; This explains why in DIII-D they noticed that up to 40% of suprathermal NBI-generated ions are lost to the edge
- a radial electric field is generated as result of this loss
- the thermal plasma responds to the radial electric field moving against the magnetic field, transversally
- the radial current of the thermal ions *partially* compensate the radial current of suprathermal ion loss

- (there is also another effect of the radial electric field? it modifies the orbits and so it influences the losses).

The rotation of plasma resulting from ion loss may be so important that *the inertial, centrifugal force becomes comparable with the pressure gradient* Especially for this case there is a possible enhancement of the rotation:

the radial currents (1) of prompt loss of energetic ions, (2) expansion of the orbit to the final banana width for the NBI ions; produce a torque both in the poloidal and the toroidal direction. The toroidal rotation produces centrifugal forces acting on the new ions, thus enhancing their radial (equatorial) motion.

4.1 A plasma of electrons and ions that advances against a neutralizing wall

4.1.1 Simplified configuration

The general motion of plasma against the wall is V_0. Conversely one says that wall advances against the plasma by $-V_0$.

The transformation

$$\begin{array}{ccl} x & \rightarrow & x' = x - V_0 t \quad \text{(horizontal)} \\ y & \rightarrow & y' = y \quad \text{(vertical)} \\ z & \rightarrow & z' = z \;\; \text{(along the confining } \mathbf{B}) \end{array}$$

and

$$\zeta \equiv x - V_0 t$$

4.1.2 Variables that will be affceted by the flow of plasma against the wall

The effect of this is:

- formation of an electric field due to charge separation. The field is

$$E_x \;,\; E_y$$

- formation of current J and alteration of the magnetic field

$$J \;\;,\; B$$

- induced displacement of plasma along the direction y,

$$V_y$$

- change in the equilibrium density

$$n_0 \rightarrow n_0 + \delta n$$

- modification of the *pressure tensor*

$$p_{xxi} \ , \ p_{xyi}$$
$$p_{xxe} \ \ , \ p_{xye}$$

  where the anisotropy is created by the different velocities along the directions x, y.

The main direction of study is

$$x \quad \text{(direction of motion)}$$

and is complemented by y (vertical).

The *fluid treatment* will include:

- equation of continuity for e and i.
- momentum equation for e and i.
- Ampere's equation to find the effect of the new electric current J on modifying the magentic field

$$\mathbf{B} \rightarrow \mathbf{B}' = \mathbf{B}_0 + \boldsymbol{\delta}\mathbf{B}$$

- Ohm's law with NO collisions, to connect E, V and B.
- Gauss law to calculate the electric field from the charge separation.

4.1.3 The equations describing the process of plasma flow against the wall

The continuity for both species e and i,

$$V_{x\alpha} n_\alpha = V_0 \left(n_\alpha - n_{0\alpha} \right)$$

The momentum conservation, at stationarity.
Along the x direction (horizontal)

$$0 = -\frac{dp_{xx\alpha}}{d\zeta} + e_\alpha n_\alpha \left(E_x + V_{y\alpha} B \right)$$

Along the vertical y direction

$$m_\alpha n_\alpha \left(-V_0 \right) \frac{dV_{y\alpha}}{d\zeta} = -\frac{dp_{xy\alpha}}{d\zeta} + e_\alpha n_\alpha \left(E_y - V_{x\alpha} B \right)$$

Equation of Ohm without resistivity

$$E_y - V_0 B = -V_0 B_0$$

Ampere's law for the modified Magnetic field

$$-\frac{dB}{d\zeta} = \mu_0 \left(n_i V_{yi} - n_e V_{ye} \right)$$

The change in the magnetic field comes from the current resulting from the different motion of electrons and ions in the vertical direction, V_{yi}, V_{ye}.

Gauss' law for the electric field

$$\frac{dE_x}{d\zeta} = \frac{1}{\varepsilon_0} |e| \left(n_i - n_e \right)$$

The effect that is expected is localized inside a distance of the order of the ion Larmor radius from the wall

$$\rho_i = \frac{v_i}{\Omega_i}$$

$$\Delta\zeta = \rho_i$$

The electron pressure is considered on these distances as istropic

$$p_{xye} = 0$$

In the equation for the momentum conservation along y, for electrons the inertia is neglected. This was the term

$$\begin{aligned} & (\mathbf{v} \cdot \boldsymbol{\nabla}) \mathbf{v} \\ \rightarrow \quad & m_e n_e (-V_0) \frac{dV_{ye}}{d\zeta} \\ & \text{negligible for electrons} \end{aligned}$$

Yushmanov Horton use the property that the *electrons are frozen in the magnetic field* and this means, for the density

$$\frac{n_e}{B} = \frac{n_{e0}}{B_0}$$

For the ions

$$\begin{aligned} n_i \quad = \quad & \frac{B}{B_0} n_{0i} + \frac{m_i}{e^2 B_0^2} \frac{d^2 p_{xxi}}{d\zeta^2} \\ & - \frac{m_i n_0}{e B_0^2} \frac{dE_x}{d\zeta} \\ & - \frac{1}{e B_0 V_0} \frac{dp_{xyi}}{d\zeta} \end{aligned}$$

The task is to calculate

$$p_{xyi}$$

with kinetic equations. Alternatively it is adopted the approximation to calculate the anisotropy using the equilibrium distribution functions, unperturbed.

The essential part of the treatment is the separation of populations

$$\begin{aligned} t &\equiv \text{thermal} \\ s &\equiv \text{suprathermal}, \;\; \rho_s \gg \rho_t \;\; \text{and} \;\; n_s \ll n_0 \end{aligned}$$

The densities and other parameters of the two components will be separated into equilibrium part "0" and perturbation due to the effects of the loss.

For thermal particles the distribution function is isotropic

$$p_{xyt} = 0$$

The anisotropy is exclusively due to the suprathermal component. Here there are two components in the pressure stress tensor due to suprathermal ions

$$\begin{aligned} p_{xyi} &\approx m_s \delta n_s V_{ys} V_0 + p_{xys} \\ &= m_s n_{0s} V_{ys} V_0 + eBV_0 \int_{\zeta}^{\infty} \delta n_s d\zeta \end{aligned}$$

The radial electric field is along x. Now, since we have an expression for the densities of electrons and ions and also the pressure anisotropy, we can use the Gauss law to obtain the electric field

$$E_x = \frac{1}{en_0} \frac{dp_{xxt}}{d\zeta} - \frac{\Omega_B B}{n_0} \int_{\zeta}^{\infty} d\zeta \; \delta n_s$$

The first term exists since p_{xxt} is the only content of the isotropic thermal pressure tensor, which is the scalar pressure of the equilibrium plasma. Certainly, there is a gradient of the scalar ion pressure directed along the radial coordinate, $x \sim \zeta$.

The vertical velocity of the thermal plasma is indiced by the motion of the plasma across the magnetic field

$$V_{yi} = \frac{\Omega_B}{n_0} \int_{\zeta}^{\infty} d\zeta \; \delta n_s$$

Both components, *thermal* and *suprathermal* have an equilibrium and a perturbed magnitude.

$$\delta n_s = n_s - n_{s0}$$

This must be calculated using the distribution function

$$f_{0s}(v_\perp)$$

where *all motion* of the suprathermal ions is represented, not only the guiding center. The *circular trajectories* of gyrating suprathermal ions lead to

$$\delta n_s = \int_{\frac{\zeta\Omega_B}{2}}^{\infty} \left[\frac{1}{\pi}\arccos\left(1 - \frac{\xi\Omega_B}{v_\perp}\right)\right] f_{0s}(v_\perp)\, dv_\perp$$

This takes into account the geometry of a circular gyration orbit that intersects a plane parpendicular on the axis $x \sim \zeta$.

The diamagnetic flow of the suprathermal ions is included.

$$n_i V_{yi} = \int_{\frac{\zeta\Omega_B}{2}}^{\infty} \left(\frac{\zeta\Omega_B}{v_\perp} - 1\right) \arccos\left(\frac{\zeta\Omega_B}{v_\perp} - 1\right) v_\perp f_{0s}(v_\perp)\, dv_\perp$$

The conclusion from the paper

the plasma rotates in the same direction as the thermal component. The velocity tangential to the wall at the location where the fraction

$$\frac{\delta n_s}{n_0}$$

of suprathermal ions has been lost is

$$V_{yi} \sim V_s \frac{\delta n_s}{n_0}$$

4.1.4 The fluid equation. General layout

This is Appendix A of the paper by **Yushmanov**.

The metric coefficients are

$$\Delta = \frac{qR}{2\pi B_T}$$

This is

$$\Delta = \frac{rB_T}{RB_\theta}\frac{R}{2\pi B_T} = \frac{1}{2\pi}\frac{r}{B_\theta}$$

If we think of circular surfaces

$$\begin{aligned}
\iint (\boldsymbol{\nabla} \times \mathbf{B}) \cdot \mathbf{dS} &= \iint \mu_0 J_T dS \\
\oint B_\theta dl &= \mu_0 I_p(r) \\
2\pi r B_\theta(r) &= \mu_0 I_p(r) \\
B_\theta(r) &= \frac{1}{r}\frac{\mu_0}{2\pi} I_p(r)
\end{aligned}$$

for a uniform current

$$\begin{aligned}
B_\theta(r) &= \frac{1}{r}\frac{\mu_0}{2\pi}\pi r^2 \bar{j} \\
\Delta &= \frac{r}{2\pi B_\theta} = \frac{1}{\mu_0 \pi \bar{j}} \sim \text{const}
\end{aligned}$$

Then this quantity is a *physical object.* It is the inverse of the rotational of the magnetic field.

Yushmanov says

$$\Delta \sim R^2$$

Further

$$\begin{aligned}
\rho &= \Delta \frac{|\nabla \psi|}{R} \\
&= qR\frac{B_P}{B_T} = \frac{rB_T}{RB_p} R \frac{B_P}{B_T} = r
\end{aligned}$$

In the simplest, circular case, $\rho \equiv r$.

$$\begin{aligned}
\eta &= \Delta \frac{|\nabla \theta|}{R} \\
&= \frac{1}{2\pi}\frac{r}{B_\theta}\frac{1}{r}\frac{1}{R} = \frac{r}{2\pi R B_\theta}
\end{aligned}$$

$$\begin{aligned}
\xi &= -\Delta^2 \frac{1}{R^2}(\nabla \psi \ \nabla \theta) \\
&\quad \text{is a tensor ?}
\end{aligned}$$

Then

$$\mathbf{v} = v_T \frac{\Delta}{R}(\nabla \psi \times \nabla \theta) + v_P \frac{D}{\rho}(\nabla \varphi \times \nabla \psi) + v_\psi \frac{\Delta}{\eta}(\nabla \theta \times \nabla \varphi)$$

With thsi expressio, the inertia term is

$$\begin{aligned}
& \nabla (n\mathbf{v}\ \mathbf{v}) \\
=\ & \mathbf{v}(\nabla n \mathbf{v}) - n(\mathbf{v}\cdot\nabla)\mathbf{v}
\end{aligned}$$

$$\begin{aligned}
& -n\left(\frac{v_T^2}{R}\nabla R + \frac{v_P^2}{\rho}\nabla \rho + \frac{v_P v_\psi}{\rho\eta}\nabla \xi + \frac{v_\psi^2}{\eta}\nabla \eta\right) \\
& +\nabla \varphi\ \nabla\cdot(nRv_T\mathbf{v}) \\
& +\nabla \theta\ \nabla\cdot\left[n\left(\rho v_P + \xi\frac{v_\psi}{\eta}\right)\mathbf{v}\right] \\
& +\nabla \psi\ \nabla\cdot\left[n\left(\xi\frac{v_P}{\rho} + \eta v_\psi\right)\mathbf{v}\right]
\end{aligned}$$

Using the same expressions one writes the equations

$$\frac{1}{\Delta}\frac{\partial}{\partial\theta}\left(\Delta n\frac{v_P}{\rho}\right)+\frac{1}{\Delta}\frac{\partial}{\partial\psi}\left(\Delta n\frac{v_\psi}{\eta}\right)=0$$

$$\begin{aligned}
&\frac{m}{R\Delta}\left[\frac{\partial}{\partial\theta}\left(\Delta n\frac{v_P}{\rho}Rv_T\right)+\frac{\partial}{\partial\psi}\left(\Delta n\frac{v_\psi}{\eta}Rv_T\right)\right]\\
= \quad & -\frac{B_T^2}{qR}\frac{\partial}{\partial\theta}\left(\frac{p_\parallel-p_\perp}{B^2}\right)\\
&+\widehat{\mathbf{e}}_T\cdot(\mathbf{F}+en\mathbf{E})\\
&+\frac{en}{2\pi}\frac{1}{R}\frac{v_\psi}{\eta}
\end{aligned}$$

$$\begin{aligned}
&\frac{m}{\Delta}\left[\frac{\partial}{\partial\theta}\left(\Delta n\frac{v_P}{\rho}Bv_\parallel\right)+\frac{\partial}{\partial\psi}\left(\Delta n\frac{v_\psi}{\eta}Bv_\parallel\right)\right]\\
&+\frac{mn}{2\pi\Delta}\frac{B^2}{B_P^2}\left(v_\perp^2\frac{1}{B_T}\frac{\partial B_T}{\partial\theta}-v_P^2\frac{1}{B}\frac{\partial B}{\partial\theta}\right)\\
= \quad & -\frac{1}{2\pi\Delta}\left[\frac{\partial}{\partial\theta}\left(\frac{p_\parallel-p_\perp}{2}\right)+B^2\frac{\partial}{\partial\theta}\left(\frac{p_\parallel-p_\perp}{2B^2}\right)\right]\\
&+\mathbf{B}\cdot(\mathbf{F}+en\mathbf{E})
\end{aligned}$$

$$-\frac{\partial p}{\partial\psi}+\frac{\mu}{\rho^2}\frac{\partial p}{\partial\theta}-en\left(\frac{\partial\phi}{\partial\psi}-\frac{\mu}{\rho^2}\frac{\partial\phi}{\partial\theta}\right)-\frac{B}{2\pi RB_P}env_\perp=0$$

The parallel and perpendicular velocities are

$$v_\parallel=\frac{\mathbf{B}}{B}\cdot\mathbf{v}=\frac{B_T}{B}v_T+\frac{B_P}{B}\left(v_P+\frac{\xi}{\rho}\frac{v_\psi}{\eta}\right)$$

$$\begin{aligned}
v_\perp &= \frac{\boldsymbol{\nabla}\psi\times\mathbf{B}}{2\pi RB_PB}\cdot\mathbf{v}\\
&= \frac{B_P}{B}v_T-\frac{B_T}{B}\left(v_P+\frac{\xi}{\rho}\frac{v_\psi}{\eta}\right)
\end{aligned}$$

The inertia terms associated with the radial velocity are ignored (in the parallel projection of the momentum).

In the radial projection of the momentum balance the velocity part of the stress tensor and the pressure anisotropy are neglected.

4.1.5 The loss cone

This is Appendix B of the same **Yushmanov Horton** paper.

The invariants

$$\begin{aligned}
\mu &= \frac{v_\perp^2}{Bv^2} \sim \text{magnetic moment (modified)} \\
v &\sim \text{energy of a particle} \\
\Psi &\sim \text{cyclotrone averaged toroidal momentum}
\end{aligned}$$

Take

$$F(\Psi) = B_T R$$

$$\Psi = \Psi_0 - 2\pi\sigma\rho_0 \left[F_0\sqrt{1-\mu B_0} - \frac{B_0}{B} F \sqrt{1-\mu B} \right]$$

Here

$$\sigma \equiv \text{sign of the parallel velocity}$$

The definition of the event in which a particle traverses the *separatrxi surface*

A particle is lost if it crosses the separatrix magnetic surface ψ_{sep} *at the location of the* X *point*

$$B = B_{sep}$$

For the moments of the distribution function, one must restrict to the region of velocity space where confined particles are.

Inside the separatrix,

$$\psi < \psi_{sep}$$

the region of integration is

$$\Xi(B > B_{sep}) \quad : \quad \int_0^\infty \int_0^{1/B} (\pm 1) - \int_{v_2}^\infty \int_{\mu_1}^{1/B} (1) - \int_{v_1}^\infty \int_{1/B_x}^{s_2} (-1)$$

and

$$\Xi(B < B_{sep}) \quad : \quad \int_0^\infty \int_0^{1/B} (\pm 1) - \int_{v_3}^\infty \int_{\mu_1}^{1/B_x} (-1) - \int_{v_1}^\infty \int_{1/B_x}^{\mu_2} (-1)$$

In the paranthesis it is the sign of the parallel velocity, σ.

In the above expressions, the first integral is

$$\begin{aligned}
& \int_0^\infty \int_0^{1/B} (\pm 1) \\
= \; & \text{integration over the full velocity space}
\end{aligned}$$

The other two are the integration over the part of the velocity space where there is *loss of particles*.

The limits

$$\mu_1 = \begin{cases} \mu_v & \text{for} \quad v < v_4 \\ 0 & \text{for} \quad v > v_4 \end{cases}$$

$$\mu_2 = \begin{cases} \mu_v & \text{for} \quad v < v_2 \\ \min\left\{\frac{1}{B_{sep}}, \frac{1}{B}\right\} & \text{for} \quad v > v_2 \end{cases}$$

where

$$\mu_v = \frac{1}{B} - \frac{1}{B}\left\{\frac{\frac{v_\psi}{v} - \sqrt{\frac{B}{B_{sep}}}\sqrt{\frac{v_\psi^2}{v^2} - \left(1 - \frac{B_{sep}}{B}\right)^2}}{1 - \frac{B_{sep}}{B}}\right\}^2$$

$$v_1 = \frac{v_\psi}{\frac{B_{sep}}{B}\sqrt{1 - \frac{B}{B_x}}} + \sqrt{1 - \frac{B_{sep}}{B_x}}$$

$$v_2 = \begin{cases} \frac{v_\psi}{\sqrt{1 - \frac{B_{sep}}{B}}} & \text{for} \quad B > B_{sep} \\ \frac{v_\psi B}{B_{sep}\sqrt{1 - \frac{B}{B_{sep}}}} & \text{for} \quad B < B_{sep} \end{cases}$$

$$v_3 = \frac{v_\psi}{\frac{B_{sep}}{B}\sqrt{1 - \frac{B}{B_x}} - \sqrt{1 - \frac{B_{sep}}{B_x}}}$$

$$v_4 = \left|\frac{v_\psi}{1 - \frac{B_{sep}}{B}}\right|$$

$$v_\psi = \frac{\psi_{sep} - \psi}{2\pi F}\frac{eB_{sep}}{m}$$

For isotropic distribution function in the incoming plasma, the correcetions to the hydrodynamic moments of the distribution function due to the loss cone can be determined

$$\delta n = -\pi \int_{v_1}^{\infty} 2\left(\mu_x - \mu_n\right) \, f(v)\, v^2 dv$$

$$\delta\left(n v_\parallel\right) = -\pi \int_{v_1}^{\infty} 2\left[-v\frac{1}{2}\left(\mu_x^2 - \mu_n^2\right)\right] \, f(v)\, v^2 dv$$

$$\delta P_\parallel = -\pi \int_{v_1}^{\infty} 2mv^2 \frac{1}{3}\left(\mu_x^3 - \mu_n^3\right) \, f(v)\, v^2 dv$$

$$\delta P_\perp = -\pi \int_{v_1}^{\infty} mv^2 \left[\mu_x - \mu_n - \frac{1}{3}\left(\mu_x^3 - \mu_n^3\right)\right] \, f(v)\, v^2 dv$$

The limits on the magnetic moments

For

$$B > B_{sep}$$

the limits for μ_n are

$$\mu_n = \begin{cases} \sqrt{1 - \mu_s B} & \text{for} \quad v_1 \le v \le v_2 \\ -\sqrt{1 - \mu_v B} & \text{for} \quad v_2 \le v \le v_4 \\ -1 & \text{for} \quad v_4 \le v \end{cases}$$

And for

$$B < B_{sep}$$

the limits for μ_n are

$$\mu_n = \begin{cases} \sqrt{1-\mu_v B} & \text{for} \quad v_1 \le v \le v_2 \\ \sqrt{1-\frac{B}{B_{sep}}} & \text{for} \quad v_2 \le v \end{cases}$$

Now the other variable , μ_x.
For

$$B > B_{sep}$$

the limits for the variable μ_x are

$$\mu_x = \sqrt{1-\frac{B}{B_x}}$$

For

$$B < B_{sep}$$

the limits are

$$\mu_x = \begin{cases} \sqrt{1-\frac{B}{B_x}} & \text{for} \quad v_1 \le v \le v_3 \\ \sqrt{1-\mu_v B} & \text{for} \quad v_3 \le v \le v_4 \\ 1 & \text{for} \quad v_4 \le v \end{cases}$$

4.2 Plasma acceleration by superthermal ion losses

Equations: continuity and toroidal and parallel components of the meoment equation.

First one defines

$$\begin{aligned} u_d &= \frac{B_T}{B_P} B v_\perp \\ &= -2\pi R B_T \left(\frac{\partial \phi}{\partial \psi} + \frac{1}{en} \frac{\partial p}{\partial \psi} \right) \end{aligned}$$

$$\begin{aligned} u_{pol} &= 2\pi\Delta \frac{v_P}{\rho} \\ &= \frac{v_P}{B_P} \end{aligned}$$

$$u_\psi = 2\pi\Delta \frac{v_\psi}{\eta}$$

$$v_T = \frac{u_d}{B_T} + u_{pol} B_T$$

$$v_\parallel = \frac{u_d}{B} + u_{pol} B$$

Then the equations, initially written for any species.
The continuity

$$\frac{\partial}{\partial\theta}(nu_P) + \frac{\partial}{\partial\psi}(nu_\psi) = 0$$

Momentum, toroidal

$$\begin{aligned} & \frac{\partial}{\partial\theta}(nu_P R v_T) + \frac{\partial}{\partial\psi}(nu_\psi R v_T) \\ = \quad & -\frac{RB_T}{m}\frac{\partial}{\partial\theta}\left(\frac{p_\parallel - p_\perp}{B^2}\right) \\ & +\frac{e}{m}n\left(U_L\Delta + \frac{u_\psi}{2\pi}\right) \\ & +\frac{2\pi R\Delta}{m}F_T \end{aligned}$$

Momentum, parallel

$$\begin{aligned} & \frac{\partial}{\partial\theta}\left(nu_P B_\parallel\right) + \frac{\partial}{\partial\psi}\left(nu_\psi B v_\parallel\right) + n\frac{u_d^2}{B_T^3}\frac{\partial B_T}{\partial\theta} - nu_P^2 B\frac{\partial B}{\partial\theta} \\ = \quad & -\frac{1}{m}\left[\frac{\partial p}{\partial\theta} + \frac{2}{3}B^{3/2}\frac{\partial}{\partial\theta}\left(\frac{p_\parallel - p_\perp}{B^{3/2}}\right)\right] \\ & +2\pi\frac{\Delta B}{m}\left(F_\parallel + enE_\parallel\right) \end{aligned}$$

where

$$U_L = \text{loop voltage}$$

Now, for electrons: neglect the inertia and the viscosity

$$0 = -en_e\left(U_L\Delta + \frac{u_{\psi e}}{2\pi}\right) + 2\pi R\Delta F_{T_e}$$

$$0 = -\frac{\partial p_e}{\partial\theta} + 2\pi\Delta B\left(F_{\parallel e} - enE_\parallel\right)$$

The ion density is represented by two components: *thermal* and *suprathermal*. Neutrality

$$n_e = n_t + n_s$$

5 The orbit of high energy ions from NBI

5.1 The loss of particles at the edge Rome McAlees Callen Fowler

See also *above*.

The paper adopts

$$j(r) = j_0 \left[1 - \left(\frac{r}{a}\right)^n\right]^p$$

It is defined a parameter

$$P \equiv \frac{qv}{\Omega_c a}$$

The invariants

$$\epsilon = \frac{mv^2}{2}$$

$$\mu = \frac{mv_\perp^2}{2B}$$

$$\cos\chi = \frac{v_\parallel}{v}$$

and the magnetic field

$$B = B_0 \frac{R_0}{R}$$

then

$$\begin{aligned}
\mu &= \left(\frac{mv^2}{2} - \frac{mv_\parallel^2}{2}\right)\frac{R}{B_0 R_0} \\
&= \frac{m}{2}v^2\left(1 - \frac{v_\parallel^2}{v^2}\right)\frac{R}{B_0 R_0} \\
&= \frac{m}{2}v^2 \sin^2\chi \frac{R}{B_0 R_0}
\end{aligned}$$

This invariant is calculated in two points

$$\mu = \frac{m}{2}v^2 \sin^2\chi \frac{R}{B_0 R_0} = \frac{m}{2}v^2 \sin^2\chi_B \frac{R_B}{B_0 R_0}$$

which is written

$$\begin{aligned}
0 &= v^2 \sin^2\chi \ R - v^2 \sin^2\chi_B \ R_B \\
0 &= v^2\left(1 - \cos^2\chi\right) \ R - v^2 \sin^2\chi_B \ R_B \\
0 &= \left(v^2 - v_\parallel^2\right) \ R - v^2 \sin^2\chi_B \ R_B
\end{aligned}$$

$$v_\parallel^2 \ R = v^2 \ R - v^2 \sin^2\chi_B \ R_B$$

we multiply with R

$$v_\parallel^2 R^2 = v^2 R\left(R - R_B \sin^2\chi_B\right)$$

$$v_\parallel R = v\sqrt{R\left(R - R_B \sin^2\chi_B\right)}$$

This expression $Rv_{\|}$ is necessary as part of the *longitudinal invariant*

$$\begin{aligned} mRv_{\varphi} &\approx mRv_{\|} \\ &= mv\sqrt{R\left(R - R_B \sin^2 \chi_B\right)} \end{aligned}$$

the longitudinal invariant is

$$-Ze\psi + mRv_{\varphi} = \text{const}$$

The part with the magnetic potential

$$\psi = -R_0 A_{\varphi}$$

The expression of the magnetic potential A_{φ} is obtained using the Ampere equation for the current density that has been adopted in the general form above, with two indices n and p.

$$\begin{aligned} \iint \mathbf{dS} \cdot \, \mathrm{curl}\, \mathbf{B} &= \mu_0 \iint dS \; j\left(r\right) \\ \oint dl \; B_{\theta}\left(r\right) &= 2\pi B_{\theta}\left(r\right) = \mu_0 \int_0^r 2\pi r dr \; j\left(r\right) \\ B_{\theta}\left(r\right) &= \frac{1}{2\pi r}\mu_0 \int_0^r 2\pi r dr \; j\left(r\right) \end{aligned}$$

and

$$B_{\theta}\left(a\right) = \frac{\mu_0 I_p}{2\pi a}$$

but

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$$

then

$$B_{\theta} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_{\varphi}\right)$$

$$\begin{aligned} A_{\varphi}\left(r\right) &= \frac{\mu_0 I_p}{2\pi} \frac{\sum_{j=0}^{p} \frac{(-1)^j \left(\frac{r}{a}\right)^{nj+2}}{(nj+2)} \frac{p!}{j!} \frac{1}{(p-j)!}}{\sum_{j=0}^{p} \frac{(-1)^j}{(nj+2)} \frac{p!}{j!} \frac{1}{(p-j)!}} \\ &= \frac{\mu_0 I_p}{2\pi} F\left(r\right) \end{aligned}$$

In this expression it has been defined a new *nondimensional* function

$$F\left(r\right)$$

With this expression for $A_\varphi(r)$ we return to the longitudinal invariant

$$ZeR_0\frac{\mu_0 I_p}{2\pi}F(r)+mv\sqrt{R\left(R-R_B\sin^2\chi_B\right)}=\text{const}$$

$$F(r)\mp\frac{2\pi mv}{ZeR_0\mu_0 I_p}\sqrt{R\left(R-R_B\sin^2\chi_B\right)}=\text{const}$$

We can use the parameter

$$P\equiv\frac{qv}{\Omega_c a}$$

and we replace

$$q=\frac{rB_\varphi}{RB_\theta}$$

where

$$B_\theta(r)=\frac{\mu_0 I_p}{2\pi a}$$

The boundary value of the safety factor is

$$\begin{aligned} q(a) &= \frac{aB_\varphi}{RB_\theta}=\frac{aB_\varphi}{R\mu_0 I_p}2\pi a \\ &= \frac{2\pi a^2 B_\varphi}{R\mu_0 I_p} \end{aligned}$$

$$\begin{aligned} P &= \frac{qv}{\frac{ZeB}{m}a}=v\frac{2\pi a^2 B_\varphi}{R\mu_0 I_p}\frac{m}{ZeBa} \\ &\approx \frac{2\pi am}{\mu_0 ZeRI_p}v \end{aligned}$$

It is introduced the ratio

$$A\equiv\frac{R}{a}$$

and

$$P=\frac{2\pi mv}{\mu_0 ZeAI_p}$$

The *longitudinal invariant* has provided the expression

$$\begin{aligned} F(r)\mp\frac{2\pi mv}{ZeR_0\mu_0 I_p}\sqrt{R\left(R-R_B\sin^2\chi_B\right)} &= \text{const} \\ F(r)\mp\frac{P}{a}\sqrt{R\left(R-R_B\sin^2\chi_B\right)} &= \text{const} \end{aligned}$$

If all lengths are normalized to a,

$$R\rightarrow\frac{R}{a}$$

the equation above becomes

$$F(r) \mp P\sqrt{R\left(R - R_B \sin^2 \chi_B\right)} = \text{const}$$

For the signs

$$\begin{array}{lll} \text{sign } - & \text{is taken for} & \mathbf{v}\cdot\mathbf{J} > 0 \\ \text{sign } + & \text{is taken for} & \mathbf{v}\cdot\mathbf{J} < 0 \end{array}$$

The largest displacement is for the barely trapped particles.

5.1.1 The limit in the velocity space of the barely trapped particles

A barely trapped particle is tangent to the equatorial plane at a point defined as *pinch point.*

This point is special, in the sense that the vertical component of the particle's drift velocity and the vertical component of the motion of the particle along the magnetic field line cancel.

If the coordinates are (x, y) where

$$\begin{aligned} x &= R + r\cos\theta \\ y &= r\sin\theta \end{aligned}$$

the *pinch* point corresponds to

$$\begin{aligned} \frac{dy}{dx} &= 0 \\ y &= 0 \end{aligned}$$

Then one returns to the equation that we have derived for the *longitudinal invariant*

$$F(r) \mp P\sqrt{R\left(R - R_B \sin^2 \chi_B\right)} = \text{const}$$

and takes the derivative dy/dx to zero. It results an implicit equation for the pinch point, x_p,

$$R_p = \frac{R_B \sin^2 \chi}{2}\left[1 + \sqrt{\frac{\left[F'(x_p)\right]^2}{\left[F'(x_p)\right]^2 - P^2}}\right]$$

The *longitudinal invariant* can be used for two different points of the orbit

- the point where the parallel velocity is zero

$$v_{\parallel} = 0$$

- the *pinch* point, as defined above

Then

$$
\begin{aligned}
& F\left(x_p\right)+P \sqrt{R\left(R-R_B \sin^2 \chi\right)} \\
= \quad & F\left(r_B\right)+P R_B|\cos \chi|
\end{aligned}
$$

The procedure of identifying the largest bananas (for barely trapped particles) consists of

- solve the equation which expresses R_p and find the pitch angle χ.
- take this pitch angle χ in the equation which results from this equality (written above) between the *longitudinal invariant* calculated in the pinch point x_p and at the tip $v_{\|}=0$ of the banana; this is to obtain a relationship between x_p (the pitch point position) and the energy, given by P.

Then

$$
f\left(x_p, P\right)=0
$$

becomes the connection that allows to find in the velocity space the coordinates χ, at a given energy P, of the limit-bananas, the barely trapped particles ("fattest bananas").

5.1.2 The limit in the velocity space of the orbits that are tangent to the limiter

The other side of the loss region in the velocity space is defined as the pitch angle χ of theose orbits that are tangent to the limiter at $x=a$.

$$
\begin{aligned}
& F(1)-P \sqrt{(A+1)\left(A+1-R_B \sin^2 \chi\right)} \\
= \quad & F\left(r_B\right)+P R_B|\cos \chi|
\end{aligned}
$$

The equation is solved analytically for $|\cos \chi|$,

$$
|\cos \chi|=\frac{-R_B K+\sqrt{R_B(A+1)\left[K^2-P^2\left(1-r_B\right)^2\right]}}{P R_B\left(1-r_B\right)}
$$

where

$$
K \equiv F(1)-F\left(r_B\right)
$$

5.1.3 The Vertex of the loss region in the velocity space

This point corresponds to the *energy* such that the barely trapped particles has orbit that is tangent to the limiter (both above conditions are met).

6 Ion orbit loss rate in tokamak (Shaing [?])

This is also in *plasma general rotation*.

To explain the H-mode in tokamaks. The loss of ions is localized in the high energy part of the distribution function, since here the ions are less collisional. Being less collisional, they have rather clear banana trajetories and in their motion they can hit the limiter.

But the expulsion of a "hot" ion from the plasma is simultaneously compensated (electrically) by the entrance of an ion from the exterior of the plasma toward the interior. This influx is driven by the ion viscosity which is essentially determined by a non-zero rotation of the plasma. It concerns the lower temperature ions, *i.e.* the ions of the Pfirsch-Schluter-plateau, more collisional. This again rises the problem of Stix : it is necessary to have the poloidal rotation + collisions in order to get a radial electric current. This current will participate in the radial component of the $\nabla \times \mathbf{B}$ equation, to compensate (together with the polarization electric current) the outflux of directly lost hot ions. Also in Roenbluth it is the current of compensation of the lost very-hot-ions at NBI.

This problem of balance of ion radial currents can be re-stated: the loss of ions is the primary process; but the plasma establishes an electric radial field andthe corresponding rotation in order to obtain, through the ion-viscosity, a radial electric current of ions which balances the outgoing ion flux.

- the *outgoing* ion orbit loss flux (in **banana regime**) , is balanced by
- incoming *viscosity driven* ion flux (plateau-Pfirsch-Schluter)

to mentain ambipolarity at the steady state.

Nota. It must be understood that the *disappearence of an ion (* whose banana orbit hits the wall or the limitator) from the plasma **should not be seen as a current directed to the wall**. On the contrary, one expects that the ion which will replace the lost ion, will come from the plasma border toward the centre. So, there is a current of response, the so-called *incoming current* or *return current.*

The effect of viscosity is separated in two in order to emphasize the two different effects:

1. "viscosity-driven flux" is the flux driven by the viscosity contributed by the particles in the regime **plateau-Pfirsch-Schluter**. Actually is the flux of ions of replacement.
2. "ion orbit loss flux" is the flux driven by the viscosity of the particles in the banana regime. (This is because the loss of ions is due to the difference in the radial drifts of the electrons and the ions, when the drifts are not too much perturbed by the collisions , *i.e.* in the banana regime). But it is not clear how the viscosity is involved in the **direct loss of banana ions to the limiter**.

When the two fluxes (outcoming and incoming) are integrated over the velocity space, they *approximately cancel* each other. The **net** ion flux cancels to order $\sqrt{m_e/m_i}$ *when E_r is determined proprely from the momentum balance equation.*

7 Plasma flows associated with ion orbit loss

7.1 Notes

Effects to be accounted for:

- neoclassical viscosity effect
- torque due to the prompt loss of ions

The physical reason for the formation of the electrostatic potential at the plasma edge is *the difference of the orbit sizes of electrons and ions*. High energy ions (suprathermal component of a tail of a Maxwellian distribution) have the largest orbit width and are lost from the largest distance from the wall : this means that the ions which are lost in a point very close to the the border belong actually to a region deeper into the plasma. Those which have greater parallel velocity but still smaller than that necessary to become transiting particles, are stoped (i.e. have banana tips) in the inner part of the tokamak plasma. They have the larger radial drift on their bananas, which also means that they belong to magnetic surfaces (in the sense that the average of the trajectory is there) deeper into the plasma. When such ion is lost through collision, a positively charged particle from a point deep in the plasma disappears.

8 NBI new ions produce current

See the file NBI

See **Cordey**.
The data depend on the machine.

From numerical simulations (**Benchmarking neutral beam injection codes 42nd EPS, Schneider et al**) several results are available

- the *birth deposition profiles* are characterized by maxima of the order

$$\dot{n}^{\max} \sim 10^{18} \left(\frac{ions}{s}\right)$$

- the *power deposition*

$$10^4 \left(\frac{W}{m^3}\right)$$

- neutral beam *current drive* is

$$50 \times 10^3 \left(\frac{A}{m^2}\right)$$

The following data are for JET

$$\begin{aligned} I_{NBI}^{equiv} &= 50 \ (A) \\ & \quad \text{equivalent current in the beam} \end{aligned}$$

$$\begin{aligned} E_{NBI}^{H^+} &= 100 \ (kV) \\ & \quad \text{electric field of acceleration} \\ & \quad \text{of the ions before being neutralized} \end{aligned}$$

The energy for a proton in this field

$$\begin{aligned} eE \times \Delta l &= \frac{m_p v^2}{2} \\ 1.6 \times 10^{-19} \ (C) \times 10^5 \ (V) \times 1 \ (m) &= \frac{1}{2} 1.6 \times 10^{-27} \ (kg) \times v^2 \end{aligned}$$

$$\begin{aligned} v^2 &= 10^{-19+27} \times 10^5 = 10^{13} \\ v &\sim 10^6 \ (m/s) \end{aligned}$$

Then

$$\begin{aligned} I &= \frac{Q}{t} = \frac{en\,(\Delta l \times S)}{\Delta t} = S \times env \\ &= S \times 1.6 \times 10^{-19} \times n \times 10^6 \left(\frac{m}{s}\right) \\ &= S \times n \times 1.6 \times 10^{-13} \\ 50 \ (A) &= S \ \left(m^2\right) \times n \left(\frac{part}{m^3}\right) \times \left(\frac{m}{s}\right) \times 1.6 \times 10^{-13} \end{aligned}$$

taking the section

$$S \sim 1 \ \left(cm^2\right) = 10^{-4} \ \left(m^2\right)$$

It results

$$n = \frac{50}{10^{-4} \times 1.6 \times 10^{-13}} = 10^{18} \left(\frac{part}{s}\right)$$

In the paper of **Rome Fowler** the data for ORMAX were

$$\begin{aligned} I^{equiv} &\sim 1.45 \ (A) \\ W_{NBI} &= 28 \ (KeV) = 2.8 \times 10^4 \times 1.6 \times 10^{-19} \ (J) \end{aligned}$$

and

$$W = \frac{mv^2}{2} \rightarrow v \sim 10^5 \ \ (m/s)$$

The equivalent current, of a beam with a cross section

$$S = 1 \ \left(cm^2\right) = 10^{-4} \ \left(m^2\right)$$

$$\begin{aligned} I^{equiv} &= env \times S \quad \left[C \frac{ions}{m^3} \frac{m}{s} \times m^2\right] \\ &= 1.6 \times 10^{-19} \times 10^5 \times 10^{-4} \times n \quad (A) \end{aligned}$$

from where the density in the volume of the beam

$$n \sim 10^{18} \ \left(\frac{ions}{m^3}\right)$$

9 Plasma flows associated with ion orbit loss

9.1 Notes

Effects to be accounted for:

- neoclassical viscosity effect
- torque due to the prompt loss of ions

The physical reason for the formation of the electrostatic potential at the plasma edge is *the difference of the orbit sizes of electrons and ions*. High energy ions (suprathermal component of a tail of a Maxwellian distribution) have the largest orbit width and are lost from the largest distance from the wall : this means that the ions which are lost in a point very close to the the border belong actually to a region deeper into the plasma. Those which have greater parallel velocity but still smaller than that necessary to become transiting particles, are stoped (i.e. have banana tips) in the inner part of the tokamak plasma. They have the larger radial drift on their bananas, which also means that they belong to magnetic surfaces (in the sense that the average of the trajectory is there) deeper into the plasma. When such ion is lost through collision, a positively charged particle from a point deep in the plasma disappears.