MST1-20-T01-AUG T01: ITER H-mode scenario operating space

Generation and saturation of the polarization electric field at ionization

1 Introduction. Physical picture

In this work we take into account the transient torque generated at the ionization of neutral atoms. For each new ion that is trapped there is a single event of displacement, from the point of ionization to the "center" of the periodic motion on banana. The ensemble of such displacements builds up a current which is significant on all duration of the ionization (of pellet, gas puff or of impurity seeding). The return current has effect on plasma rotation, on formation of transport barriers and may lead to "pellet enhanced performance". We examine this process in connection with charge separation, polarization, radial electric field and radial current. These processes are relevant in the formation of transport barriers and obtainment of the high confinement regimes in tokamak. It is admitted that the transport barriers exist as sheared poloidal rotation of the plasma in narrow layers, in particular close to the LCFS in the case of H mode. Both rotations (poloidal and toroidal) exist in tokamak, but the efficiency in suppression of turbulent fluctuations is substantially different: the toroidal rotation can even sustain turbulent instabilities while the sheared poloidal rotation modifies both the linear phase (and ability to extract free energy) and the radial extension of the eddies, reducing the diffusion coefficient. However the poloidal plasma rotation is subject to a strong damping mechanism (the magnetic pumping) and it also carries an inertial factor. For the poloidal plasma rotation to exist it is necessary a mechanism that continuously sustain it, like a radial electric current. A torque acting in the poloidal direction might be able to overcome the magnetic damping and ensure the turbulence reduction or even suppression on the region of sheared rotation. This means formation of a transport barrier.

We have considered two such mechanisms: (1) the NBI ions that are born on trapped orbits; (2) ionization of neutral gas with which plasma is feeded continuously (gas puff, pellets, recycling). This can also be extended to low Z impurities that are sometimes used to control the ELMs.

We have calculated the radial current that is produced in plasma by the new ions in the transitory phase in which they move to place themselves on the periodic banana orbits. This phase is unique but the total radial current is substantial.

The main problem that we have formulated and analysed in detail (but still being studied) is the saturation of the polarization electric field. It is formulated as follows.

Consider an ionization event which generates an electron and an ion. The electron has a small radial drift (because of its small mass) and will remain close to the magnetic surface. The ion has a much larger radial displacement, which can be seen as between the point where it is born to the "center of its banana orbit" which is the average of the all positions on the banana. This displacement is a radial current and is the unique event which consists of current, since after that the motion (on banana) is periodic. However this current is repeated for all the ionization events (leading to trapped ions) and the total magnitude can be substantial. By the conservation of charge the background plasma must respond with an opposite (return) current and this will produce a torque $\mathbf{j}^{return} \times \mathbf{B}$ acting on plasma and producing rotation.

In the same time we have a separation of charge: the electron and the ions, initially occupying the same position at ionization are now separated by a finite distance on the equatorial plane (due to banana symmetry). There is an electric field that results, a polarization. The effect is a drift $E \times B$ is added to the neoclassical drift of particles.

A natural question can be formulated: since the process of ionization is continuous (or acting on a very long time scale) the charge separation seems to continuously increase and the electric field (polarization) will increase too. The question is what is a mechanism that can saturate the growth of the electric field, preventing it to reach extremely high magnitude.

We can think to a new ion that must move to its "center of banana" against the already present polarization electric field. On one hand the ion motion will also contain the $E \times B$ drift in the neoclassical equations. On the other hand the problem is no more only kinematic (as the neoclassical equations are $d\mathbf{x}/dt = \mathbf{v}_{drift}^{neo}$) but also energetic. Since the ion is constrained to settle on the banana orbit (a pure geometrical effect), it will have to spend energy to overcome the obstacle of the electric field. This energy is taken from the initial energy of the ions and this will modify its trajectory. In particular the electric field with large spatial variation will produce the squeezing factor for the banana. But we cannot expect that the only effect of the new ion moving against the polarization electric field is the squeezing factor. This by itself cannot provide saturation.

We have to consider the other processes in which is involved the polarization electric field. One of them is the rotation drift $E \times B$ of the bulk plasma. This is subject to collisional friction. Then the polarization electric field resulting from charge separation at ionization will only increase until the rotation is saturated by viscosity. This is the suggestion made by Honda et al. without however providing an analytical framework describing the establishment of stationary equilibrium.

2 Radial electric field and radial current (fluid approach)

We review here the fluid approach to the polarization process (Honda et al.). The magnetic field

$$\mathbf{B} = \nabla\varphi \times \nabla\psi + I\nabla\varphi$$

and Ampere's law

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

the density of current is *total*, includes imposed current and plasma response (polarization).

Surface averge of the radial projection

$$\langle \mathbf{j} \cdot \nabla\psi \rangle = \varepsilon_0 \frac{\partial}{\partial t}\left(\left\langle |\nabla\psi|^2 \right\rangle \frac{\partial\phi}{\partial\psi} \right)$$

The radial current (surface-averaged) is connected with the time variation of the radial electric field.

The determination of the radial current will be done in *fluid* formulation.

The current crossing the surface (radial) is determined by friction forces acting in toroidal momentum balance.

$$\begin{aligned} m_a n_a \frac{d\mathbf{u}_a}{dt} &= -\nabla p_a - \nabla \cdot \boldsymbol{\pi}_a \\ &\quad + e_a n_a \mathbf{E} + \mathbf{e}_a n_a \mathbf{u}_a \times \mathbf{B} \\ &\quad + \mathbf{R}_a \end{aligned}$$

There will be two operations on this momentum conservation equation.

One is the projection on the *toroidal* direction $\nabla\varphi$, followed by surface average.

The other is *parallel* projection (on the magnetic field direction $\mathbf{B}$) followed by the surface average.

Even if the distinction between these two directions ($\nabla\varphi$ and $\mathbf{B}$) seems weak the first operation emphasize the radial current while the second cannot.

The momentum equation is projected on the toroidal direction, $R^2\nabla\varphi\cdot$, plus $\langle\rangle$

$$\begin{aligned} \left\langle m_a n_a \, R^2 \nabla\varphi \cdot \frac{d\mathbf{u}_a}{dt} \right\rangle &= -\left\langle R^2 \nabla\varphi \cdot \nabla \cdot \boldsymbol{\pi}_a \right\rangle \\ &\quad + \left\langle e_a n_a \, R^2 \nabla\varphi \cdot \mathbf{E} \right\rangle \\ &\quad + \left\langle e_a n_a \, R^2 \nabla\varphi \cdot (\mathbf{u}_a \times \mathbf{B}) \right\rangle \text{ this will give radial current } j_r \\ &\quad + \left\langle R^2 \nabla\varphi \cdot \mathbf{R}_a \right\rangle \end{aligned}$$

The *toroidal* projection of the momentum equation contains the term $\mathbf{j} \times \mathbf{B}$ from which the radial current will be extracted.

The *parallel* projection of the momentum equation, that will be used below, does not keep the product $\mathbf{B} \cdot (\mathbf{u}_a \times \mathbf{B})$ which is zero. The parallel projection is obtained by scalar-multiplication with $\mathbf{B}$, surface averaging and summing over species a,

$$\sum_{a=e,i} \left\langle \mathbf{B} \cdot m_a n_a \frac{\partial \mathbf{u}_a}{\partial t} \right\rangle = - \sum_{a=e,i} \langle \mathbf{B} \cdot \boldsymbol{\nabla} \cdot \boldsymbol{\pi}_a \rangle + \sum_{a=e,i} \left\langle \mathbf{B} \cdot \mathbf{R}_a^{non-C} \right\rangle$$

(where $\mathbf{E}$ and the Coulombian collisional friction $\mathbf{R}^C$ disappear by summation over species).

We note that

$$\begin{aligned} \mathbf{B} \times \boldsymbol{\nabla} \varphi &= B \, \frac{1}{R} \frac{B_\theta}{B} \widehat{\mathbf{e}}_{\boldsymbol{\nabla} \psi} \\ &= \frac{1}{R^2} R B_\theta \widehat{\mathbf{e}}_{\boldsymbol{\nabla} \psi} \end{aligned}$$

and the last three factors are

$$R B_\theta \widehat{\mathbf{e}}_{\boldsymbol{\nabla} \psi} = \boldsymbol{\nabla} \psi$$

Then the third term in the RHS (the Lorentz force term) is

$$\begin{aligned} \left\langle e_a n_a \; R^2 \boldsymbol{\nabla} \varphi \cdot (\mathbf{u}_a \times \mathbf{B}) \right\rangle &= \left\langle e_a n_a \; R^2 \; \mathbf{u}_a \cdot (\mathbf{B} \times \boldsymbol{\nabla} \varphi) \right\rangle \\ &= \left\langle e_a n_a R^2 \; \frac{1}{R^2} \mathbf{u}_a \cdot \boldsymbol{\nabla} \psi \right\rangle \\ &= \langle \mathbf{j}_a \cdot \boldsymbol{\nabla} \psi \rangle \end{aligned}$$

This term from the *toroidal* $\left(R^2 \boldsymbol{\nabla} \varphi \cdot \right)$ projection of the momentum for species a is the surface average of the radial current.

The convective part of the time derivative have no contribution after toroidal projection and surface averaging

$$\left\langle R^2 \boldsymbol{\nabla} \varphi \cdot (\mathbf{u} \cdot \boldsymbol{\nabla}) \, \mathbf{u} \right\rangle = 0$$

The demonstration is given by **Hirshman** and mentioned by **Hinton Rosenbluth** also detailed by **Honda**; it involves the periodicity. This will reduce the total time derivative to the explicit derivative to time $\partial / \partial t$ and it is this term that will represent the polarization: the time change of the velocity is due to charge separation.

We separate the main object of interest, the surface-averaged radial current of the species a. The radial current is connected (by its combination with $\mathbf{B}$)

with every force (change of momentum in time) which acts in toroidal direction

$$
\begin{aligned}
\langle \mathbf{j}_a \cdot \boldsymbol{\nabla}\psi \rangle &= -\left\langle e_a n_a \; R^2 \boldsymbol{\nabla}\varphi \cdot \mathbf{E} \right\rangle \text{ toroidal electric field} \\
&\quad -\left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \mathbf{R}_a \right\rangle \quad \text{friction, toroidal} \\
&\quad +\left\langle m_a n_a \; R^2 \boldsymbol{\nabla}\varphi \cdot \frac{\partial \mathbf{u}_a}{\partial t} \right\rangle \text{ polarization} \\
&\quad +\left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla} \cdot \boldsymbol{\pi}_a \right\rangle \text{ viscosity}
\end{aligned}
$$

We **note** the third term, $\left\langle m_a n_a \; R^2 \boldsymbol{\nabla}\varphi \cdot \frac{\partial \mathbf{u}_a}{\partial t} \right\rangle$. This term shows that a time variation of the toroidal flow $\boldsymbol{\nabla}\varphi \cdot \frac{\partial \mathbf{u}_a}{\partial t}$ is related to a *radial current*. This is normal, in the connection $\mathbf{j}_r$ →torque→toroidal acceleration, since the radial current × the poloidal magnetic field $\mathbf{j}_r \times \mathbf{B}_\theta$ is a force which injects momentum in every unit of time in the heavy component, ions, of plasma. The toroidal plasma flow is *accelerated* ($\partial/\partial t \neq 0$) when there is a radial current.

The reversed relation is less usual: if a factor generates *acceleration* of the toroidal flow (like in NBI momentum transfer), then a radial current should arise.

The resistive friction $\mathbf{R}$ is separated in two parts: Coulombian and non-Coulombian (turbulent)

$$
\mathbf{R}_a = \mathbf{R}_a^C + \mathbf{R}_a^{non-C}
$$

The coulombian part is coupled with the electric field

$$
\begin{aligned}
\langle \mathbf{j}_a \cdot \boldsymbol{\nabla}\psi \rangle &= -\left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \left(\mathbf{R}_a^C + e_a n_a \mathbf{E} \right) \right\rangle \text{ resistive flux} \\
&\quad +\left\langle m_a n_a \; R^2 \boldsymbol{\nabla}\varphi \cdot \frac{\partial \mathbf{u}_a}{\partial t} \right\rangle \text{ polarization flux} \\
&\quad +\left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \left(\boldsymbol{\nabla} \cdot \boldsymbol{\pi}_a - \mathbf{R}_a^{non-C} \right) \right\rangle \text{ orthogonal conduction flux}
\end{aligned}
$$

Regarding the first term: (1) the coulombian collisions conserve momentum; then the sum over species will give zero. (2) In addition we assume neutrality of plasma and this makes the terms with toroidal electric field to disappear when summing over species.

$$
\sum_{e,i} \left\langle R^2 \boldsymbol{\nabla}\varphi \cdot \left(\mathbf{R}_a^C + e_a n_a \mathbf{E} \right) \right\rangle \rightarrow 0.
$$

The term that comes from the change of momentum orthogonal on surfaces contains the *pressure stress*.

$$
\boldsymbol{\pi}_a = \boldsymbol{\pi}_a^{(1)} + \boldsymbol{\pi}_a^{(2)}
$$

where Chow Goldberger Low form

$$
\boldsymbol{\pi}_a^{(1)} = \left(p_{a\parallel} - p_{a\perp} \right) \left(\widehat{\mathbf{n}} \; \widehat{\mathbf{n}} - \frac{1}{3} \mathbf{I} \right)
$$

For this part the operations ($R^2\boldsymbol{\nabla}\varphi\cdot$ followed by surface averaging) will give *zero*

$$\left\langle R^2\boldsymbol{\nabla}\varphi\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_a^{(1)}\right\rangle = 0$$

The demonstration is in **Hirshman Sigmar**.

Only the higher orders $\boldsymbol{\pi}_a^{(2)}$ can contribute to exchange of toroidal momentum in the perpendicular direction on surface (what remains after surface averaging).

Then one can sum over species

$$\begin{aligned}\langle\mathbf{j}\cdot\boldsymbol{\nabla}\psi\rangle &= \sum_{a=e,i}\left[\left\langle m_a n_a\ R\frac{\partial u_{a\varphi}}{\partial t}\right\rangle \quad \text{polarization flux}\right.\\ &\quad+\left\langle R^2\boldsymbol{\nabla}\varphi\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_a^{(2)}\right\rangle \quad \text{perpendicular viscosity}\\ &\quad\left.-\left\langle R^2\boldsymbol{\nabla}\varphi\cdot\mathbf{R}_a^{non-C}\right\rangle\right] \quad \text{cross-field resistive flux (small)}\end{aligned}$$

It is now introduced a new object of interest

$$\begin{aligned}&\text{polarization current}\\ \equiv\ &(\text{def})\ \mathbf{j}_a^{pol}\end{aligned}$$

Honda assumes that the *polarization* current is connected with the time variation (acceleration) of the toroidal rotation

$$\begin{aligned}&\left\langle\mathbf{j}_a^{pol}\cdot\boldsymbol{\nabla}\psi\right\rangle\\ =\ &\sum_{a=e,i}\left\langle m_a n_a\ R\frac{\partial u_{a\varphi}}{\partial t}\right\rangle\end{aligned}$$

The general form of the velocity has components that are

- toroidal $\sim\boldsymbol{\nabla}\varphi$,
- parallel, $\sim\mathbf{B}$,

$$\begin{aligned}\mathbf{u}_a &= \omega_a\ R^2\boldsymbol{\nabla}\varphi+\widehat{u}_{a\theta}\ \mathbf{B}\\ &\quad(\text{toroidal})+(\text{parallel})\end{aligned}$$

$$\begin{aligned}\omega_a &= -\frac{1}{e_a n_a}\frac{\partial p_a}{\partial\psi}-\frac{\partial\phi}{\partial\psi}\\ &\quad\theta-\text{"diamagnetic" plus electric}\\ &\quad\text{only depend on }\psi\end{aligned}$$

We call θ-*diamagnetic* the combination between $\frac{\partial p}{\partial \psi} \sim \widehat{\mathbf{e}}_r$ (radial, since $\frac{\partial}{\partial \psi} = \frac{1}{|\boldsymbol{\nabla}\psi|}\frac{\partial}{\partial r} = \frac{1}{RB_\theta}\frac{\partial}{\partial r}$) and B_θ (poloidal) and is directed in the toroidal direction $\sim \boldsymbol{\nabla}\varphi$.

Projecting $\mathbf{u}_a$ on toroidal direction

$$u_{a\varphi} = \omega_a R + \frac{I}{R}\widehat{u}_{a\theta}$$

Projecting $\mathbf{u}_a$ on parallel direction

$$u_{a\parallel} = \omega_a \frac{I}{B} + \widehat{u}_{a\theta}\ B$$

NOTE that a generic form of the parallel velocity is adopted, with the *poloidal* term left unknown, as KB and determined by a constraint equation. **END.**

We will need the surface average of $Ru_{a\varphi}$, the toroidal flow associated to the radial polarization current.

Then we multiply the first by R (equivalent to multiplying by $R^2\boldsymbol{\nabla}\varphi\cdot$) and average

$$\langle Ru_{a\varphi}\rangle = \omega_a \left\langle R^2\right\rangle + I\left\langle \widehat{u}_{a\theta}\right\rangle$$

To eliminate $\widehat{u}_{a\theta}$ we first multiply the parallel equation to B and average

$$\left\langle Bu_{a\parallel}\right\rangle = \omega_a I + \left\langle \widehat{u}_{a\theta}B^2\right\rangle$$

Now we have to take into account that

$$\left\langle \widehat{u}_{a\theta}\right\rangle = \widehat{u}_{a\theta}$$

then

$$\frac{\left\langle Bu_{a\parallel}\right\rangle}{\left\langle B^2\right\rangle} = \omega_a \frac{I}{\left\langle B^2\right\rangle} + \left\langle \widehat{u}_{a\theta}\right\rangle$$

Multiply this equation by $-I$ and add the two equations

$$\langle Ru_{a\varphi}\rangle - I\frac{\left\langle Bu_{a\parallel}\right\rangle}{\left\langle B^2\right\rangle} = \omega_a \left[\left\langle R^2\right\rangle - I^2\frac{1}{\left\langle B^2\right\rangle}\right]$$

Since we expect the *neoclassical polarization* to be derived we separate

$$B^2 = \frac{I^2}{R^2} + B_\theta^2$$

and take surface average

$$\left\langle B^2\right\rangle = I^2\left\langle \frac{1}{R^2}\right\rangle + \left\langle B_\theta^2\right\rangle$$

next we rewrite to find the ratio

$$\frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle} = 1 - I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle$$

In the expression that multiplies ω_a we separate a quantity that is known

$$\begin{aligned} & \left\langle R^2\right\rangle - I^2 \frac{1}{\left\langle B^2\right\rangle} \\ = & \left\langle R^2\right\rangle - I^2 \frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}\frac{1}{\left\langle B_\theta^2\right\rangle} \end{aligned}$$

Let us add and subtract a term

$$\left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle$$

$$\begin{aligned} & \left\langle R^2\right\rangle - \left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle + \left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle - I^2 \frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}\frac{1}{\left\langle B_\theta^2\right\rangle} \\ = & \left\langle R^2\right\rangle - \left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle + I^2 \left\langle R^2\right\rangle \frac{1}{\left\langle B_\theta^2\right\rangle}\left[\frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle - \frac{1}{\left\langle R^2\right\rangle}\frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}\right] \\ = & \left\langle R^2\right\rangle - \left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle + \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}\frac{I^2}{\left\langle B_\theta^2\right\rangle}\left[\left\langle \frac{1}{R^2}\right\rangle - \frac{1}{\left\langle R^2\right\rangle}\right] \end{aligned}$$

For the last factor in the last line we introduce the notation

$$\frac{I^2}{\left\langle B_\theta^2\right\rangle}\left[\left\langle \frac{1}{R^2}\right\rangle - \frac{1}{\left\langle R^2\right\rangle}\right] \equiv 2\widehat{q}^2$$

and the last line takes the form

$$\left\langle R^2\right\rangle - \left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle + \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle} 2\widehat{q}^2$$

One can prove that the first two terms can be combined as

$$\left\langle R^2\right\rangle - \left\langle R^2\right\rangle I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle = \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}$$

To prove that this is correct, one starts by using the relation derived before

$$\frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle} = 1 - I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle$$

Here we isolate

$$I^2 \frac{1}{\left\langle B^2\right\rangle}\left\langle \frac{1}{R^2}\right\rangle = 1 - \frac{\left\langle B_\theta^2\right\rangle}{\left\langle B^2\right\rangle}$$

and replace in the LHS

$$
\begin{aligned}
& \left\langle R^2\right\rangle - \left\langle R^2\right\rangle I^2 \frac{1}{\langle B^2\rangle} \left\langle \frac{1}{R^2} \right\rangle \\
= & \left\langle R^2\right\rangle - \left\langle R^2\right\rangle \left(1 - \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle}\right) \\
= & \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle} \quad \text{OK}
\end{aligned}
$$

Then indeed the term in paranthesis multiplying ω_a is

$$
\begin{aligned}
\left\langle R^2\right\rangle - I^2 \frac{1}{\langle B^2\rangle} &= \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle} + \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle} 2\widehat{q}^2 \\
&= \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle} \left(1 + 2\widehat{q}^2\right)
\end{aligned}
$$

Replacing

$$
\begin{aligned}
\langle R u_{a\varphi}\rangle &= I \frac{\left\langle B u_{a\|}\right\rangle}{\langle B^2\rangle} + \omega_a \left[\left\langle R^2\right\rangle - I^2 \frac{1}{\langle B^2\rangle}\right] \\
&= I \frac{\left\langle B u_{a\|}\right\rangle}{\langle B^2\rangle} + \omega_a \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle} \left(1 + 2\widehat{q}^2\right)
\end{aligned}
$$

We return to the expression of the polarization current. The polarization current is the part with acceleration of the toroidal flow, $\partial/\partial t$

$$
\begin{aligned}
& \left\langle \mathbf{j}_a^{pol} \cdot \boldsymbol{\nabla}\psi \right\rangle = (\text{def}) \\
= & \sum_{a=e,i} \left\langle m_a n_a \, R \frac{\partial u_{a\varphi}}{\partial t} \right\rangle
\end{aligned}
$$

Now we express the *toroidal velocity* $u_{a\varphi}$ by *parallel velocity* $u_{a\|}$, using the equation $\langle R u_{a\varphi}\rangle - I \frac{\left\langle B u_{a\|}\right\rangle}{\langle B^2\rangle} = \omega_a \left[\left\langle R^2\right\rangle - I^2 \frac{1}{\langle B^2\rangle}\right]$ which we have commented above

$$
\langle R u_{a\varphi}\rangle (\text{toroidal}) - I \frac{\left\langle B u_{a\|}\right\rangle (\text{parallel})}{\langle B^2\rangle} = \omega_a \left[\left\langle R^2\right\rangle - I^2 \frac{1}{\langle B^2\rangle}\right]
$$

Then from the definition of the polarization current

$$
\begin{aligned}
& \left\langle \mathbf{j}_a^{pol} \cdot \boldsymbol{\nabla}\psi \right\rangle \overset{\text{def}}{=} \sum_{a=e,i} \left\langle m_a n_a \, R \frac{\partial u_{a\varphi}}{\partial t} \right\rangle \\
= & \sum_{a=e,i} m_a n_a \left[I \frac{1}{\langle B^2\rangle} \frac{\partial \left\langle B u_{a\|}\right\rangle}{\partial t} \right. \\
& \left. + \left\langle R^2\right\rangle \frac{\left\langle B_\theta^2\right\rangle}{\langle B^2\rangle} \left(1 + 2\widehat{q}^2\right) \frac{\partial \omega_a}{\partial t} \right]
\end{aligned}
$$

The first term $I\frac{1}{\langle B^2\rangle}\frac{\partial\langle Bu_{a\|}\rangle}{\partial t}$ introduces the time variation of the *parallel* velocity $u_{a\|}$, the *acceleration* of the parallel velocity. This is because **Honda** associates the polarization *current* with the effect of the torque = injection of moment per unit time in the toroidal direction i.e. acceleration of the *toroidal rotation*.

This is different compared to **Novakovskii Liu Sagdeev Rosenbluth** who consider the *poloidal rotation damping* and coupling with Geodesic Acoustic Modes.

In conclusion in this (fluid) approach it was derived the connection between the radial current leading to charge separation (polarization) and the time variation of the toroidal and parallel flows. It does not yet respond to our question (which is the saturation of the charge separation?) but shows that the surface averaging the forces only cross-field resistive flux (for example from turbulence) and high order perpendicular viscosity can balance a radial current (charge separation) without indefinite acceleration of the toroidal flow $\partial u_a/\partial t = 0$.

3 The polarization (kinetic approach)

It is considered a fast temporal variation of the radial electric field. This is accompanied by a change in the neoclassical prolarization.

For the *barely circulating ions*, it is possible to calculate the radial polarization current (**Novakovski**). It is assumed that the radial electric field has a time variation which can be linearized

$$E_r = E_{r0} + \left(\frac{\partial E_r}{\partial t}\right)t$$

accordingly the electric (poloidal) velocity now has time variation

$$v_E = v_{E0} + \left(\frac{\partial v_E}{\partial t}\right)t \quad \text{(in the poloidal direction)}$$

The particles that are considered are

$$\begin{array}{rcl} & & \text{trapped ions} \\ \text{with } v_\| & \ll & v_\perp \end{array}$$

The equation for the poloidal motion

$$r\frac{d\theta}{dt} = v_E + v_\|\frac{B_\theta}{B_T}$$

where

$$\frac{B_\theta}{B_T} = \frac{\varepsilon}{q} \equiv \Theta$$

is the factor that projects the parallel direction on the poloidal direction. Integrating on time

$$r\theta(t) = r\theta_0 + \left(v_{E0} + v_\parallel \frac{B_\theta}{B_T}\right) t + \frac{1}{2}\left(\frac{\partial v_E}{\partial t}\right) t^2$$

This is the (short-time) evolution of the poloidal position of an ion.

The radial velocity is the radial component of the *drift* of the guiding center

$$v_r = \mathbf{v}_D \cdot \widehat{\mathbf{e}}_r = v_D \sin\theta$$

$$v_D = \frac{1}{\Omega} \frac{v_\perp^2/2 + v_\parallel^2}{R}$$

We assume *circulating* particle but with very small *parallel* velocity,

$$v_\parallel \ll v_\perp$$

$$v_{D,r} \approx \frac{1}{\Omega} \frac{\frac{v_\perp^2}{2}}{R} \sin\theta = \frac{1}{\Omega} \frac{1}{R} \frac{m v_\perp^2}{2B} \frac{B}{m} \sin\theta = \frac{1}{\Omega R} \frac{\mu B}{m} \sin\theta$$

(**note** that here $\mu = \frac{m v_\perp^2}{2B}$) and

$$v_r(t) = \frac{1}{\Omega R} \frac{\mu B}{m} \sin[\theta(t)]$$

and the average over a period is

$$\begin{aligned} \langle v_r \rangle &= \frac{1}{2} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt\ v_r(t) \\ &= \frac{1}{\Omega R} \frac{\mu B}{m} \frac{1}{2} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt\ \sin[\theta(t)] \end{aligned}$$

Now we replace the equation for $\theta(t)$ and expand the function $\sin\theta$ for small argument

$$\begin{aligned} \sin[\theta(t)] &= \sin\left[\theta_0 + \frac{1}{r}\left(v_{E0} + v_\parallel \frac{B_\theta}{B_T}\right) t + \frac{1}{2}\left(\frac{\partial v_E}{\partial t}\right) t^2\right] \\ &\approx \sin\theta_0 \\ &\quad + \cos\theta_0 \left[\frac{1}{r}\left(v_{E0} + v_\parallel \frac{B_\theta}{B_T}\right) t + \frac{1}{r}\frac{1}{2}\left(\frac{\partial v_E}{\partial t}\right) t^2\right] \end{aligned}$$

and the integrations over the period of poloidal circuit gives

$$\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt\ \sin\theta_0 = \text{constant}$$

This constant must be zero since there is no constant averaged radial velocity.

$$\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt\ \cos\theta_0 \frac{1}{r}\left(v_{E0} + v_\parallel \frac{B_\theta}{B_T}\right) t = \cos\theta_0 \frac{1}{r}\left(v_{E0} + v_\parallel \frac{B_\theta}{B_T}\right) \frac{1}{\tau}\left[\frac{\tau^2}{2}\right]_{-\tau/2}^{\tau/2} = 0$$

This reflects the periodicity, as expected if we work with closed bananas.

The difference arises if there is a change *in time* of the radial electric field $E_r(t)$, as assumed. Equivalently, there is a change in time of the electric velocity, which is poloidal, $\partial v_E/\partial t$. If the time scale of the variation of the radial electric field (equivalently of v_E) is comparable with the bounce time, then the particle will move on half of the banana with a drift velocity $v_{D,r}$ and on the second half of the banana with a *different* $v_{D,r}$, which gives a different rate of time evolution of the poloidal angle $\theta(t)$. This is retained as the second term in the expansion of $\sin[\theta(t)]$. It is *this* difference, which is produced by the time variation of the rate of change of the angle $\theta(t)$, that gives in the end a non-zero radial *average* velocity $\langle v_r \rangle$. For a purely periodic banana this should not exist. The motion on a banana is no more periodic when the radial electric field has very fast variation.

We have

$$\frac{1}{\tau}\int_{-\tau/2}^{\tau/2} dt\ \cos\theta_0 \frac{1}{r}\frac{1}{2}\left(\frac{\partial v_E}{\partial t}\right) t^2 = \frac{1}{24}\cos\theta_0 \frac{1}{r}\left(\frac{\partial v_E}{\partial t}\right)\tau^2$$

The radial velocity averaged over a period of the motion on banana is at this moment

$$\begin{aligned}\langle v_r \rangle &= \frac{1}{\Omega R}\frac{\mu B}{m}\frac{1}{2}\frac{1}{\tau}\int_{-\tau/2}^{\tau/2} dt\ \sin[\theta(t)] \\ &= \frac{1}{\Omega R}\frac{\mu B}{m}\frac{1}{48}\cos\theta_0 \frac{1}{r}\left(\frac{\partial v_E}{\partial t}\right)\tau^2\end{aligned}$$

Now we take

$$\begin{aligned}\tau &\equiv \text{time of a period} \\ &\sim \frac{\text{connection length } 2\pi q R}{\text{parallel velocity } v_{\parallel}} \\ &= \frac{2\pi q R}{|v_{\parallel}|}\end{aligned}$$

It is necessary to define the regime by few parameters.

$$\begin{aligned}\widehat{\nu} &= def = \frac{r\ \nu_{ii}}{\Theta v_{th,i}} \\ &= \frac{\text{freq. of ion collisions}}{\text{freq. of ion transit with poloidal velocity } \Theta v_{th,i} \text{ on poloidal circle}}\end{aligned}$$

the same formula is written

$$\begin{aligned}\widehat{\nu} &= \frac{r\ \nu_{ii}}{\Theta v_{th,i}} \\ &= \frac{r\ \nu_{ii}}{\frac{\varepsilon}{q} v_{th,i}} \\ &= \frac{\nu_{ii}}{v_{th,i}/(qR)}\end{aligned}$$

This parameter compares the collisions with the motion on poloidal direction.

[**note** in **Hazeltine Ware electrostatic trapping** the parameter is $\widehat{\nu} = r\frac{\nu_{e,i}}{\frac{B_\theta}{B}v_{th,e,i}}$, identical].

The parallel velocity of the ions is taken at the limit where the effective ion collision frequency is equal with the parallel transit frequency

$$\nu_{eff} = \frac{v_\parallel}{qR}$$

where by definition

$$\nu_{eff} \overset{def}{=} \nu_{ii}\frac{v_{th,i}^2}{v_\parallel^2}$$

We combine the two expression of ν_{eff} and further use the expression for $\widehat{\nu}$

$$\begin{aligned}\nu_{ii}\frac{v_{th,i}^2}{v_\parallel^2} &= \frac{v_\parallel}{qR} \quad \text{or} \\ \frac{\nu_{ii}}{v_{th,i}/(qR)} &= \frac{v_\parallel^3}{v_{th,i}^3} \\ \widehat{\nu} &= \frac{v_\parallel^3}{v_{th,i}^3}\end{aligned}$$

from where we derive

$$\frac{v_\parallel}{v_{th,i}} = \widehat{\nu}^{1/3}$$

and we replace $v_\parallel$ with the expression in terms of thermal ion velocity and the effective collision parameter $\widehat{\nu}$

$$v_\parallel = v_{i,th}\widehat{\nu}^{1/3}$$

This is an important connection. It is not universal, it actually is a *choice*.

The connection is between the parallel velocity $v_\parallel$ and the thermal ion velocity, mediated by the parameter which measures the collisions relative to poloidal bounce.

Then the square of the period time τ is

$$\begin{aligned}\tau^2 &= \frac{(2\pi)^2 q^2 R^2}{v_\parallel^2} \\ &= \frac{(2\pi)^2 q^2 R^2}{v_{i,th}^2}\widehat{\nu}^{-2/3}\end{aligned}$$

and

$$
\begin{aligned}
\langle v_r \rangle &= \frac{1}{\Omega R} \frac{\mu B}{m} \frac{1}{48} \cos\theta_0 \frac{1}{r} \left(\frac{\partial v_E}{\partial t} \right) \tau^2 \\
&= \frac{1}{\Omega R} \frac{\mu B}{m} \frac{1}{48} \cos\theta_0 \frac{1}{r} \left(\frac{\partial v_E}{\partial t} \right) \frac{(2\pi)^2 q^2 R^2}{v_{i,th}^2} \widehat{\nu}^{-2/3} \\
&= \frac{(2\pi)^2}{48} \frac{q^2}{\varepsilon \Omega} \frac{\mu B}{m} \cos\theta_0 \left(\frac{\partial v_E}{\partial t} \right) \widehat{\nu}^{-2/3}
\end{aligned}
$$

To calculate the radial current two steps are necessary:
- take the fraction of the particles that are in this regime
- integrate over the positions θ_0. Actually, the parameter θ_0 appears in the magnitude of the magnetic field $B = B_0 (1 - \varepsilon \cos\theta_0)$ and this, in turn, appears in the expression of the magnetic momentum $\mu = v_\perp^2 / (2B)$. Then to integrate over the Maxwell distribution of the variable $v_\perp$ we can equivalently integrate over the variable θ_0 for fixed μ.

The fraction of particles is

$$
\sim \frac{v_\parallel}{v_{i,th}}
$$

and this is

$$
\text{fraction of particles} \sim \widehat{\nu}^{1/3}
$$

When we multiply the average radial velocity by this factor

$$
\begin{aligned}
& \widehat{\nu}^{1/3} \times \widehat{\nu}^{-2/3} \\
= & \widehat{\nu}^{-1/3}
\end{aligned}
$$

we get a dependence of the effective collisional parameter as $\widehat{\nu}^{-1/3}$ which will be found in the final expression.

The Maxwellian in velocity space for *deep trapped* (very small $v_\parallel$) is

$$
\begin{aligned}
f_M &= N \exp\left(-\frac{mv^2}{2T} \right) \\
&= N \exp\left(-\frac{m\left(v_\parallel^2 + v_\perp^2\right)}{2T} \right) \sim N \exp\left(-\frac{mv_\perp^2}{2T} \right) \\
&= N \exp\left(-\frac{\mu B}{T} \right) = N \exp\left(-\frac{\mu B_0}{T(1+\varepsilon\cos\theta)} \right) \\
&\approx N \exp\left[-\frac{\mu B_0}{T} (1 - \varepsilon \cos\theta) \right]
\end{aligned}
$$

We use this velocity integration to suppress the indeterminancy given by the presence of θ_0 in the radial current.

$$
\frac{\partial v_E}{\partial t} = \frac{1}{B} \left(\frac{\partial E_r}{\partial t} \right)
$$

Then the radial electric current (surface-averaged) induced by the time variation of the radial electric field is

$$\langle j_r \rangle \approx \left(1 + q^2 + \widehat{\nu}^{-1/3} q^2\right) \frac{m}{B^2} \left(\frac{\partial E_r}{\partial t}\right)$$

In this formula, 1 is the standard polarization term. The second term is the neoclassical polarization term due to ions with comparable parallel and perpendicular velocities, $v_\parallel \approx v_\perp$.

The neoclassical polarization radial current due to radial excursions of the banana (*trapped* particles) when there is fast time variation of the radial electric field, is

$$j_r^{bananas} \approx \varepsilon^{3/2} \frac{c^2}{v_{A\theta}^2} \left(\frac{\partial E_r}{\partial t}\right)$$

We **note** the substantial difference relative to the classical polarization

$$\frac{c^2}{v_A^2} \rightarrow \frac{c^2}{v_{A\theta}^2}$$

and a factor that reflects the fraction of population

$$\varepsilon^{3/2}$$

See **Hinton Robertson**.

The equations are

$$\frac{d\mathbf{r}}{dt} = v_\parallel \widehat{\mathbf{n}} + \mathbf{v}_E + \mathbf{v}_D$$

$$\begin{aligned}
\frac{dv_\parallel}{dt} &= \left(-\frac{v_\perp^2}{2}\widehat{\mathbf{n}} + v_\parallel \mathbf{v}_E\right) \cdot \boldsymbol{\nabla} \ln B \\
\frac{d}{dt}\left(\frac{v_\perp^2}{2}\right) &= \frac{v_\perp^2}{2}\left(v_\parallel \widehat{\mathbf{n}} + \mathbf{v}_E\right) \cdot \boldsymbol{\nabla} \ln B
\end{aligned}$$

The *drift velocity* is

$$\mathbf{v}_D = \frac{1}{\Omega_{ci}} \left(\frac{v_\perp^2}{2} + v_\parallel^2\right) \widehat{\mathbf{n}} \times \boldsymbol{\nabla} \ln B + \frac{1}{\Omega_{ci}} \widehat{\mathbf{n}} \times \frac{\partial \mathbf{v}_E}{\partial t}$$

Comparing with previous expressions of the drift velocity v_D we note that there is an additional term, which gives the effect of the fast time variation of the radial electric field $\frac{\partial \mathbf{v}_E}{\partial t}$, like in transitions with rapid change of toroidal and/or poloidal rotation velocity.

We note however that the time variation of the electric drift velocity has the following effect on the drift:

we suppose that

$$\frac{\partial E_r}{\partial t} \sim \widehat{\mathbf{e}}_r$$

exists due to the polarization effect related to the forced increase of the poloidal velocity

$$\frac{\partial v_\theta}{\partial t} \rightarrow \frac{\partial E_r}{\partial t}$$

Then $\mathbf{v}_E$ will increase in two possible directions

$$\frac{\partial \mathbf{v}_E}{\partial t} \sim \frac{1}{B}\left(\frac{\partial E_r}{\partial t}\widehat{\mathbf{e}}_r \times \mathbf{B}_\theta + \frac{\partial E_r}{\partial t}\widehat{\mathbf{e}}_r \times \mathbf{B}_T\right)$$

Then the terms mentioned by **Novakovskii** is

$$\begin{aligned}\frac{1}{\Omega_{ci}}\widehat{\mathbf{n}} \times \frac{\partial \mathbf{v}_E}{\partial t} \quad &\sim \quad \frac{1}{\Omega_{ci}}\widehat{\mathbf{n}} \times \left(\frac{\partial E_r}{\partial t}\widehat{\mathbf{e}}_r \times \mathbf{B}_\theta\right) \quad \text{almost zero} \\ &\quad + \frac{1}{\Omega_{ci}}\widehat{\mathbf{n}} \times \left(\frac{\partial E_r}{\partial t}\widehat{\mathbf{e}}_r \times \mathbf{B}_T\right) \quad \text{radial}\end{aligned}$$

Therefore *none* of these contributions is aligned along the toroidal direction, giving a *drift* of the particle population in the toroidal direction.

It seems that a treatment based on the equations of motion of the particles governed by the invariants

$$E, \mu$$

cannot give us a drift of the bananas in the toroidal direction.

Here again the drift-kinetic equation is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} f + \frac{dv_\parallel}{dt}\frac{\partial f}{\partial v_\parallel} + \frac{d\left(v_\perp^2/2\right)}{dt}\frac{\partial f}{\partial\left(v_\perp^2/2\right)} = St\left(f\right)$$

The paper of **Novakovskii** wants to solve the problem of decay of poloidal rotation in the *plateau* regime.

Then the Drift-Kinetic equation is solved by perturbations.

Zero + order 1 + order 2 are necessary.

In the zeroth order

$$\left(v_\parallel \frac{B_\theta}{B_T} + v_E\right)\frac{\partial f_0}{r\partial\theta} - St\left(f_0\right) = 0$$

which gives a Maxwellian function *possibly* shifted in the parallel direction by a velocity U_0.

$$f_0 = \left(1 - \frac{mv_\parallel U_0}{T}\right) f_M$$

for

$$f_M = \frac{n}{\left(2\pi T/m\right)^{3/2}} \exp\left[-\frac{m\left(v_\parallel^2 + v_\perp^2\right)}{2T}\right]$$

NOTE
We remark the combination

$$\begin{aligned} & v_{\parallel}\frac{B_{\theta}}{B_T}+v_E \\ \sim\quad & \Theta\left(v_{\parallel}+\frac{v_E}{\Theta}\right) \\ \approx\quad & 0 \qquad \text{(since the paranthesis is } \sim 0) \end{aligned}$$

The combination $v_{\parallel}\frac{B_{\theta}}{B_T}+v_E$ is the *poloidal velocity*.

It is composed of the *projection* of the parallel velocity on θ, using Θ, plus the poloidal velocity due to the radial electric field.

The first term $\left(v_{\parallel}\frac{B_{\theta}}{B_T}+v_E\right)\frac{\partial f_0}{r\partial\theta}$ is the convection of the distribution function f_0 in the poloidal direction.

It is either veru small or zero.

If it exists, it is balanced by collisions.

END

Nothing at this moment suggests there can be a velocity U in the *parallel* direction, *i.e.* along the magnetic field lines. But the equation for f_0 allows it and since we know it can exist, it is introduced at this point.

Note that the velocity along the magnetic field lines comes from a shift in the parallel *particle* velocity, as

$$\begin{aligned} & -\frac{\left(v_{\parallel}-U_0\right)^2}{2T/m} \\ = \quad & -\frac{v_{\parallel}^2}{2T/m}-\frac{2v_{\parallel}U_0}{2T/m}-\frac{U_0^2}{2T/m} \end{aligned}$$

and the last term is much less than 1 since the flow with velocity U_0 is slower than the *thermal* velocity.

Then a substitution is made for the distribution function to extract a rigid body rotation

$$\begin{aligned} f \quad = \quad & f_0+\varepsilon\left(\frac{mv_{\parallel}U_0}{T}\right)f_M \\ & +\widetilde{f} \end{aligned}$$

Then the Drift-kinetic equation to order ε^2 gives

$$\begin{aligned} & \frac{\partial\widetilde{f}}{\partial t}+\left(v_E+v_{\parallel}\frac{B_{\theta}}{B_T}\right)\frac{\partial\widetilde{f}}{r\partial\theta}-St\left(\widetilde{f}\right) \\ = \quad & \frac{\sin\theta}{R}\frac{m\left(v_{\parallel}^2+\frac{v_{\perp}^2}{2}\right)}{T}Wf_M \end{aligned}$$

where W is a velocity in the poloidal direction

$$W \equiv v_E + v_{*n} + U_0 \frac{B_\theta}{B_T} + v_{*T} \left[\frac{m\left(v_\parallel^2 + v_\perp^2\right)}{2T} - \frac{3}{2} \right]$$

$$v_{*n} \equiv \frac{T}{eB} \frac{d}{dr} \ln n$$
$$v_{*T} \equiv \frac{T}{eB} \frac{d}{dr} \ln T$$

COMMENT

Then W is

W = poloidal velocity due to the radial electric field
+diamagnetic-density velocity (poloidal)
+diamagnetic-temperature velocity (poloidal)
+poloidal projection of the parallel flow velocity U_0

The term $\frac{\sin\theta}{R} \frac{m\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)}{T} W f_M$ comes from

$$\mathbf{v} \cdot \boldsymbol{\nabla} f_M$$

and reflects the convective variation of the Maxwellian with the flow velocity that exists in the plasma. The space variation of the Maxwellian f_M is *RADIAL* and the operator $\boldsymbol{\nabla}$ will be reduced to radial derivative

$$\boldsymbol{\nabla} \rightarrow \frac{\partial}{\partial r} = \frac{d}{dr}$$

Then *which is the plasma velocity that will take advantage of this radial variation of the equilibrium distribution function*? It is the particle's drift velocty $\mathbf{v}_D$ which will act along the minor radius v_{Dr}.

The term is actually

$$v_{Dr} \frac{df_M}{dr}$$

This will become the inhomogeneous term that drives a variation of the distribution function, asking therefore for the existence of a f_1.

But what can the correction do to balance this *radial convective variation of the equilibrium distribution Maxwellian function* ?

The correction f_1 actually has variation in the magnetic surface.

It will again be question of a *convective variation*, which means that there is a poloidal velocity that will advect the function f_1 along its variation.

This poloidal advection of the correction $f_1(\theta)$ will compensate for the *radial* variation of the equilibrium distribution function.

The collision operator is adopted as

$$St(f) = -\nu_{eff} f$$

$$\nu_{eff} = \frac{v_{th}^2}{v_{\parallel}^2} \nu_c$$

Regarding the application of this analysis to the case of a **fast time variation** of the radial electric field (for fast transients of the poloidal or toroidal rotation) the range of validity is established by **Novakovskii et al** by choosing

$$\frac{v_{Th}}{qR} \approx \nu_{eff} \gg \frac{\partial}{\partial t}$$

which means: the frequency of the *bounce* of the trapped particle, $v_{Th}/(qR)$, comparable with the frequency of collisions ν_{eff} is much higher than the frequency associated to the variation of the radial electric field, $\partial/\partial t$. Then during the variation of the radial electric field, which is slow, the trapped particle makes many bounces.

Then a new small parameter has been identified and the distribution function can be expanded in a series. The distribution function is only the *correction* to the shifted Maxwellian, *i.e.* the function $\widetilde{f}$ and the series is

$$\widetilde{f} = f_1 + f_2 + ...$$

Separately and related this time to the spatial variation of the distribution function, it is *considered* the variation in the magnetic surface, *i.e.* the dependence of the distribution functions f_i of the *poloidal angle* θ:

$$f_i = \sum_{\sigma=\pm 1} f_{i\sigma} \exp(i\sigma\theta)$$

Then we get the solution for the first order correction $f_{1\sigma}$ as

$$f_{1\sigma} = -\frac{\varepsilon \frac{\left(v_{\perp}^2/2 + v_{\parallel}^2\right)}{2T/m} W}{v_{\parallel}(B_\theta/B_T) + v_E - \iota\sigma\nu_{eff}} f_M$$

where

$$\iota = -\frac{1}{q}$$

note that usually is $-\frac{2\pi}{\iota} = q$.

$$\sigma = \pm 1$$

NOTE that the denominator

$$\frac{1}{v_{\parallel}\left(B_{\theta}/B_{T}\right)+v_{E}-\iota\sigma\nu_{eff}}$$

is not singular only due to collisions. The collisions prevent the resonance.

END.

Using the first order in the small parameter

$$\frac{\partial/\partial t}{v_{Th}/\left(qR\right)}\ll 1$$

and the ordering

$$\begin{aligned} v_{E} &\ll v_{Th}\frac{B_{\theta}}{B_{T}} \\ v_{E} &\ll v_{th}\Theta \end{aligned}$$

the second order contribution to the distribution function $\widetilde{f}$ is obtained from the differential equation

$$\frac{\partial f_{1}}{\partial t}+v_{\parallel}\frac{B_{\theta}}{B_{T}}\frac{\partial f_{2}}{r\partial\theta}=-\nu_{eff}f_{2}$$

(we **note** that $v_{\parallel}\frac{B_{\theta}}{B_{T}}=v_{\theta}$) from which a solution is obtained

$$f_{2\sigma}=-\iota\frac{\varepsilon\sigma r\frac{\left(v_{\perp}^{2}/2+v_{\parallel}^{2}\right)}{2T/m}}{\left[v_{\parallel}\left(B_{\theta}/B_{T}\right)-\iota\sigma r\nu_{eff}\right]^{2}}f_{M}\ \frac{\partial v_{E}}{\partial t}$$

COMMENT

The second order correction is obtained from the balance with the *time variation* of the first order perturbation, which means

$$f_{2}\sim\frac{\partial f_{1}}{\partial t}$$

This is because the expression for the first order correction f_1 contains the factor W which was derived from the radial variation of the equilibrium distribution function $f_0 \sim f_M$.

The factor W contains the *electric potential* ϕ that has radial variation

$$\phi=\phi\left(r\right)$$

BUT it also has a time variation

$$\phi=\phi\left(r,t\right)$$

since the decay of poloidal rotation consists of the chage of the radial electric field that produces the torque.

$$\begin{array}{rcl} & & \text{torque to stop poloidal rotation} \\ & \downarrow & \\ E_r & = & E_r\left(t\right) \\ & \sim & \text{decay of } v_\theta = \dfrac{E_r}{B_T} \end{array}$$

Here it is substituted

$$\nu_{eff} \approx \frac{v_{Th}^2}{v_\parallel^2} \nu_i$$

and the second order contribution to the distribution function becomes (omiting the term with $\cos\theta$)

$$f_2 = \frac{\left(\frac{v_\parallel}{v_{Th}}\right)^6 - \widehat{\nu}^2}{\left[\left(\frac{v_\parallel}{v_{Th}}\right)^6 + \widehat{\nu}^2\right]^2} \sin\theta \; \frac{\varepsilon r}{\left(v_{Th}\frac{B_\theta}{B_T}\right)^2} \frac{\left(v_\perp^2/2 + v_\parallel^2\right)}{T/m} \left(\frac{v_\parallel}{v_{Th}}\right)^4 f_M \; \frac{\partial v_E}{\partial t}$$

where it is noted later

$$\frac{v_\parallel}{v_{Th}} \equiv x$$

and

$$\widehat{\nu} \equiv \frac{r}{v_{Th}\frac{B_\theta}{B_T}} \nu_i$$

which is related to the standard neoclassical collisional parameter ν_* by

$$\begin{array}{rcl} \widehat{\nu} & = & \varepsilon^{3/2}\nu_* \\ & = & \text{plateau collisionality parameter} \end{array}$$

In order to calculate the magnetic damping of the poloidal rotation it is necessary to start from the radial electric current which on a magnetic surface must have the average equal to zero

$$\langle j_r \rangle = 0$$

The radial fluxes are considered

$$\langle nV_r \rangle = \frac{1}{2\pi} \int_0^{2\pi} d^3v \; d\theta \; v_r \left(1 + \varepsilon\cos\theta\right) \; f$$

where

$$d^3v = 2\pi dv_\parallel d\left(\frac{v_\perp^2}{2}\right)$$

The radial component of the *particle drift* velocity is

$$v_r = -\frac{1}{\Omega_c}\left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\frac{\sin\theta}{R} + \frac{1}{\Omega_c}\frac{\partial v_E}{\partial t}$$

In the integral for the radial particle fluxes one substitutes the *ion* distribution function

$$f = f_0 + f_1 + f_2 + ...$$

the expansion in the small parameter representing the ratio between the characteristic frequency of the variation of the radial electric field and the bounce frequency.

$$\begin{aligned} & \int d^3v d\theta \left[f_0 \frac{\partial v_E}{\partial t} - f_2 \left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\frac{\sin\theta}{R}\right] \\ = & \int d^3v d\theta \ f_1 \left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\frac{\sin\theta}{R} \end{aligned}$$

If the plateau collision parameter is small

$$\widehat{\nu} \ll 1$$

then the distribution function in order 1 can be approximated

$$f_1 \approx -\pi q \frac{\frac{v_\perp^2}{2} + v_\parallel^2}{T/m} W \ f_M \ \delta\left(v_\parallel\right)\sin\theta$$

NOTE that the most important contribution to the distribution function, *i.e.* f_1 comes from the *barely trapped ions*. This means that $v_\parallel \approx 0$. **End.**

Then the equation for the poloidal velocity becomes

$$\left(1 + q^2\Lambda\right)\frac{\partial v_E}{\partial t} = -\nu_{MP}\left(v_E + v_{*n} + U_0\frac{B_\theta}{B_T} + \frac{3}{2}v_{*T}\right)$$

where the rate of magnetic pumping damping is

$$\nu_{MP} = \sqrt{\frac{\pi}{2}}\frac{q v_{Th}}{R}$$

and

$$\Lambda \equiv \frac{3}{2} + \Xi\widehat{\nu}^{-1/3}$$

$$\Xi \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} dx \ x^4 \frac{x^6 - 1}{\left(x^6+1\right)^2}\exp\left(-\widehat{\nu}^{2/3}\frac{x^2}{2}\right)$$

We note the significant difference between the fluid approach and the kinetic one.

In the fluid treatment the objective was to connect the electric current flowing in radial direction (averaged on surface) with the toroidal acceleration of the plasma. If there is an extrinsic mechanism of charge separation (e.g. at ionization) then the radial current exists and the toroidal flow is saturated ($\partial/\partial t \rightarrow 0$) if only the viscosity absorbs the momentum.

In the kinetic treatment of the fast changes (damping) of the poloidal rotation, the radial current (averaged on surface) must be zero. This constraint induces the connection between the convection of the θ-dependent distribution function (this reflects the neoclassical drift of particles) and the time variation of the first order distribution function. We obtain an equation for the poloidal velocity. Since no source has been assumed, the equation is of damping.

4 The saturation by viscosity friction of the radial polarization field

NOTE

We compare the result of **Fong Hahm**

$$\mathbf{j}_{pol} \approx \frac{16}{3\pi\sqrt{2}}\ \varepsilon_a^{3/2}\ \frac{n_i m_i c^2}{B_\theta^2}\ \frac{\partial \mathbf{E}_r}{\partial t}$$

or

$$\mathbf{j}_{pol} \approx \frac{16}{3\pi\sqrt{2}}\ \varepsilon_a^{3/2}\ \frac{c^2}{v_{A\theta}^2}\frac{\partial \mathbf{E}_r}{\partial t}$$

and **Novakovskii Liu Sagdeev Galeev** and **Novakovskii Liu Sagdeev Rosenbluth**. This is for the time evolution of the poloidal rotation generated by the non-ambipolarity of the radial diffusion fluxes of ions (large) and electrons (small)

$$1.52\ nmq^2\sqrt{\frac{S}{\varepsilon}}\frac{\partial V_E}{\partial t} = -\frac{F_\parallel}{\Theta}$$

We remind that $\Theta = \frac{B_\theta}{B_T} = \tan$ (small angle made by the line with toroidal direction). Then $1/\Theta$ is tan (large $\sim \pi/2$ angle between the line and the poloidal direction)

In the LHS we have the time variation of the momentum carried by the electric velocity. This is a force.

In RHS we have a force which is parallel $F_\parallel$ and is projected on the poloidal direction $\times\frac{B_T}{B_\theta}$.

Consider

$$\begin{aligned}
\mathbf{j}_{pol}^{FH} &\approx \frac{16}{3\pi\sqrt{2}}\ \varepsilon_a^{3/2}\ \frac{n_i m_i c^2}{B_\theta^2}\ \frac{\partial \mathbf{E}_r}{\partial t} \\
&= \frac{16}{3\pi\sqrt{2}}\left(\frac{r}{R}\right)^2\frac{1}{\sqrt{\varepsilon}}\frac{B_T^2}{B_\theta^2}\frac{1}{B_T^2}n_i m_i\frac{\partial V_E}{\partial t}B \\
&\approx \frac{16}{3\pi\sqrt{2}}\ q^2\frac{1}{\sqrt{\varepsilon}}\frac{n_i m_i}{B}\frac{\partial V_E}{\partial t} \\
&\approx \frac{1}{B}\times 1.2\ n_i m_i q^2\frac{1}{\sqrt{\varepsilon}}\frac{\partial V_E}{\partial t}
\end{aligned}$$

If we take the squeezing factor

$$S \sim 1$$

then the main parameter content of the two formulas is identical, with only a factor $1/B$. We have

$$\begin{aligned} J_{pol}^{FH} &= \frac{1}{B} \times \left(-\frac{F_{\parallel}}{\Theta} \right)^{NLSG} \\ &\sim \frac{F_{\parallel}}{B_{\theta}} \end{aligned}$$

this means, if the type of the formula is

$$\frac{\widehat{\mathbf{n}}_{\theta} \times \mathbf{F}_{\parallel}}{B_{\theta}} = \text{flux}$$

that the *polarization current* is *RADIAL*.

The formula for the parallel *VISCOSITY* force is

$$\begin{aligned} F_{\parallel} &= (\boldsymbol{\nabla}\pi)_{\parallel} \\ &= 0.37\ n_i m_i \nu \sqrt{\frac{S}{\varepsilon}} \left[\frac{V_E}{\Theta} + U_{\parallel} + \frac{1}{\Theta S} (U_p - 1.17\ U_T) \right] \\ &= \frac{1}{\Theta} 0.37\ n_i m_i \nu \sqrt{\frac{S}{\varepsilon}} \left[V_E + \Theta U_{\parallel} + \frac{1}{S} (U_p - 1.17\ U_T) \right] \end{aligned}$$

where

$$U_p = \frac{T/m}{(eB/m)} \frac{d \ln (nT)}{dr}$$

$$U_T = \frac{T/m}{(eB/m)} \frac{d \ln T}{dr}$$

The velocity V_E is perpendicular on B and the two *diamagnetic* velocities U_p and U_T are also perpendicular. Then $U_{\parallel}$ must be projected using a factor

$$\frac{1}{\Theta} = \frac{B_T}{B_{\theta}}$$

The full equation contains now poloidal rotation velocities and is then projected again on the parallel direction.

END.

5 The radial current from NBI ions and from the *alpha* particles

The NBI ions that are trapped on banana orbits will produce a transient current, as explained. This current is at the origin of the charge separation and therefore of the radial electric field of polarization. This ase ilustrate clearly the problem formulated initially.

The radial current from the NBI ions generates a return current of the bulk ion plasma. The Ampere's law is

$$0 = \mu_0 \left\langle \left(\mathbf{j}^{return} + \mathbf{j}^{NBI}\right) \cdot \boldsymbol{\nabla}\psi \right\rangle + \frac{1}{c}\frac{\partial}{\partial t} \left\langle \mathbf{E} \cdot \boldsymbol{\nabla}\psi \right\rangle$$

It is well known that this leads to the plasma permitivity

$$\varepsilon_\perp = \varepsilon_0 \left(1 + \frac{c^2}{v_A^2}\right)$$

which is much larger than ε_0. The term with $\mathbf{E}$ can be neglected and the equilibrium simply consists of the return current being equal and opposite to the ionization current.

$$\left\langle \mathbf{j}^{NBI} \cdot \boldsymbol{\nabla}\psi \right\rangle = -\left\langle \mathbf{j}^{return} \cdot \boldsymbol{\nabla}\psi \right\rangle$$

The toroidal momentum conservation

$$m_i n_i \frac{\partial}{\partial t} \left\langle R u_\varphi \right\rangle = -\left\langle R\widehat{\mathbf{e}}_\varphi \cdot \left(\boldsymbol{\nabla} \cdot \boldsymbol{\pi}_i\right)\right\rangle - \left\langle \mathbf{j}^{return} \cdot \boldsymbol{\nabla}\psi \right\rangle - \left\langle R\widehat{\mathbf{e}}_\varphi \cdot \mathbf{F}_{\alpha e} \right\rangle$$

As shown in the fluid treatment

$$\left\langle \mathbf{j}_\alpha^{friction} \cdot \boldsymbol{\nabla}\psi \right\rangle = -\left\langle R\widehat{\mathbf{e}}_\varphi \cdot \mathbf{F}_{\alpha e}^{(1)} \right\rangle$$

with (1) having the meaning of the force calculated with the first order kinetic function. Part of the second and the third term cancel. What remains is part of the second term, but with the current generated by the NBI ions (transient)

$$\left\langle \mathbf{j}^{transit} \cdot \boldsymbol{\nabla}\psi \right\rangle = -\left|e\right| I^2 \frac{v_0^2}{2} \mathcal{I} \frac{\partial}{\partial \psi} \left[\dot{n}\left(\psi, 0\right) e^{-2\nu_s t}\right]$$

and after calculation of $\mathcal{I}$,

$$\left\langle \mathbf{j}^{transit} \cdot \boldsymbol{\nabla}\psi \right\rangle = -0.54 \times \varepsilon^{3/2} \left|e\right| I^2 \frac{v_0^2}{\Omega_{ci}^2} \frac{\partial}{\partial \psi} \left[\dot{n}\left(\psi, 0\right) e^{-2\nu_s t}\right]$$

It is used the approximation

$$\left\langle R\widehat{\mathbf{e}}_\varphi \cdot \left(\boldsymbol{\nabla} \cdot \boldsymbol{\pi}_i\right)\right\rangle = \frac{m_i n_i \left\langle R u_\varphi \right\rangle}{\tau_\varphi}$$

and the solution is obtained after integration on time

$$\left\langle R u_\varphi \right\rangle = C \frac{\partial}{\partial \psi} \left\{ \frac{\exp\left(-2\nu_s t\right) - \exp\left(-\frac{t}{\tau_\varphi}\right)}{\frac{1}{\tau_\varphi} - 2\nu_s} \dot{n}_\alpha \left(\psi, 0\right) \right\}$$

with C a constant.

A viscosity ν_s has been introduced to represent the decay of the velocity of NBI ions (**Rosenbluth Hinton**). We note that the result is the toroidal velocity decays together with the NBI ions velocity.

6 Conclusions

Using current analytical models (fluid and kinetic) we have examined the problem of connection between the radial current (or charge separation) and the flows in plasma. The objective was to identify the mechanisms that are able to absorb the momentum generated by the radial polarization current such that it is reached saturation of the charge separation. Since the charge separation occur in the transitory phase of the ionization events and since the ionization is not limited in time there should be somehow reached a steady state where the charge separation stops growing.

We note that the two approaches are able to indicate in a general manner that the viscosity is the mechanism that can limit the growth of the radial electric field resulting from charge separation. The steady state corresponds actually to the equality between the ionization current and the return current, with possibility of neglecting the polarization electric field. However we still do not have a definite expression for the asymptotic radial electric field.

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