

MST1-20-T04-AUG

T04: Natural stationary no-ELM regime compatibility with ITER & DEMO

Intermittent transport events caused by the toroidal/poloidal sheared flow interactions

1 Introduction

In recent experiments it has been noticed that the confinement is perturbed randomly by events of high loss of particles and energy. They occur in both L and H mode and can lead to loss representing 50% of the total $E \times B_\varphi$ radial transport. Such events are convective and originate in the region near the Last Closed Flux Surface (LCFS) or even in SOL. This is not yet the Edge Localized Modes that occur in the H mode and are almost periodic (diether, type III and especially type I). Plasma formations (called IPO = Intermittent Plasma Objects in **Boedo** et al.) are ejected into the Scrape Off Layer region carrying large amounts of particles and energy. Their dynamics is similar to that of blobs, an old and still not completely solved problem. We notice that the experimental observation of such blobs ejected from plasma indicates that some of them occur in pair ("hole" and blob of pressure perturbation) and this suggests that they may be vortices. Then the study of formation and propagation of monopolar and dipolar vortices may be useful in the static or rotating edge plasma. We focus in this work to a different possible source of transport intermittency.

We adopt the view that the random transients (intermittency) should be seen as the signature of the competition between the sheared poloidal rotation and the sheared toroidal direction with opposite effects on transport.

Both poloidal and toroidal rotation are observed in the edge plasma. Their effect on the turbulent fluctuations is different. The poloidal rotation act even at the linear level of a drift instability, shifting the eigenmode from the resonant surface and reducing the efficiency of extraction of free energy from the background gradients (**Rosenbluth**). More important, the sheared poloidal rotation tears apart the radially elongated eddies of the turbulence and reduces the diffusion coefficient. This may be at the origin of the formation of transport barriers, including the H mode. The toroidal rotation has less damping effect on the turbulence and can even be a source of turbulence. The sheared toroidal rotation can evolve into the Kelvin Helmholtz instability (Rosenbluth Catto Liu 1973).

The poloidal rotation has two obstacles to overcome (1) the poloidal inertia ($1 + 2q^2$) factor. In the presence of a radial electric field E_r the poloidal term of $\mathbf{E} \times \mathbf{B}$ becomes comparable with the toroidal term although B_θ/B_T is small; (2) the damping due to the Transit Time Magnetic Pumping. The poloidal velocity can be sustained from two sources: (A) the radial electric field caused by polarization (for example the direct loss of NBI ions); (B) the Reynolds stress generated by the instabilities produced by the sheared toroidal flow.

2 Model for intermittent loss of the transport barrier

The transport barrier is created by poloidal flow with sheared velocity. The transport barrier exists as long as there is sheared poloidal velocity. There is also toroidal flow with sheared toroidal velocity. In conditions that are usually met in tokamak edge it is known to sustain turbulence of instabilities like Kelvin Helmholtz. The Reynolds stress of instabilities sustained by the toroidal flow has a component which can transfer momentum to poloidal rotation. This transfer of momentum from toroidal to poloidal rotation helps formation of the transport barrier. In support of this picture, one notes that in experiment it has been seen that at the L to H transition the toroidal flow is reduced while the poloidal flow increases.

Due to the transport barrier there is increase of the gradients of the plasma parameters. The KH instability has less condition to grow and the turbulence sustained by the sheared toroidal flow reduces the Reynolds stress transfer to the poloidal rotation. With a weaker drive the poloidal rotation is damped by magnetic pumping and the transport barrier is lost.

The process is periodic if there is an external drive for toroidal rotation, like NBI.

3 Sheared parallel flow and the Kelvin Helmholtz instability

In **Rosenbluth Catto Liu 1973 sheared parallel** it is described the physical mechanism of the drift wave and of the KH instability when there is a radial gradient of the parallel velocity.

The geometry is defined in a slab model: the main magnetic field in the z direction, $\mathbf{B}_0 = \hat{\mathbf{e}}_z B_0$, the transversal plane is: $x \equiv$ radial, $y \equiv$ poloidal. There is a flow parallel to the magnetic field $\mathbf{U}(x) = U(x) \hat{\mathbf{e}}_z$ with shear $U' \equiv \frac{dU}{dx} > 0$. This is the local representation of the sheared toroidal flow observed in tokamak. The equilibrium density $N(x)$ has a gradient in the radial direction, $N' \equiv \frac{dN}{dx} > 0$.

The first assumption is the existence of an electric field of a electrostatic perturbation that propagates in the plane (y, z) . This is $\mathbf{E} = -\nabla\phi$ and we adopt a Fourier representation, $\phi \sim \exp(ik_y y + ik_z z - i\omega t)$, equivalently $E_z = -\frac{\partial}{\partial z}\phi = -ik_z\phi$ and $E_y = -\frac{\partial}{\partial y}\phi = -ik_y\phi$,

$$\mathbf{E} = -(ik_y\phi)\hat{\mathbf{e}}_y - (ik_z\phi)\hat{\mathbf{e}}_z$$

\mathbf{E} produces a perturbation of the charge distribution. Due to the electric field the *ions* will move along the magnetic field line \mathbf{B}_0 to neutralize the charge. Then they will get a velocity $m_i \frac{\delta u_z^{(1)}}{\delta t} = |e| E_z = -|e| \frac{\partial \phi}{\partial z} = -|e| ik_z \phi$ from

where

$$\delta u_z^{(1)} = -\frac{|e|}{m_i} i k_z \phi \delta t$$

This is (acceleration $-\frac{|e|}{m_i} i k_z \phi$) \times (time interval δt). The field \mathbf{E} of the wave accelerates the ions along the z direction and the ion density is perturbed along the line. On the other hand the electric field of the wave and the static magnetic field induce a radial drift $\mathbf{E} \times \mathbf{B}_0$ of the ions

$$u_x = \frac{-i k_y \phi}{B_0}$$

This $E \times B$ drift is directed radially. It then will involve both equilibrium gradients $\frac{dU}{dx}$ and $\frac{dN}{dx}$. The radial velocity transports to a reference x_0 plane ions from the smaller or higher x positions, where the density $N(x)$ is higher or smaller than $N(x_0)$. Then the ions that have moved parallel to the magnetic line due to the wave field are replaced with ions from lower x or from higher x . Assume

$$\frac{k_z}{k_y} > 0$$

(the wave propagates along \mathbf{B} and along the positive poloidal direction y (or increasing θ). Then the ions that have been accelerated along \mathbf{B} in the plane x_0 and have left their position along z will be replaced by ions coming from deeper region, *i.e.* from lower radial planes $x < x_0$ where the toroidal flow has a velocity $U(x)$ that is smaller than $U(x_0)$ since we have assumed $U' > 0$. Then ions at x_0 undergo another change in velocity in the time δt ,

$$\delta u_z^{(2)} = -(u_x \delta t) \frac{dU}{dx}$$

Therefore we have *two* changes of velocity of the ions: $\delta u_z^{(1)}$ (acceleration in the wave potential) and $\delta u_z^{(2)}$ (convection to x_0 from deeper x where are slower ions, or convection from higher x where are faster ions).

The Kelvin Helmholtz instability exists is

$$\left| \delta u_z^{(2)} \right| > \left| \delta u_z^{(1)} \right|$$

or

$$\frac{U'}{\Omega_i} > \frac{k_z}{k_y}$$

which means that the ions always get higher velocity by the combination of wave acceleration and sheared toroidal velocity.

The gradient of density $N(x)$ can have a stabilizing effect.

To examine this it is adopted the following regime of parameters

$$\frac{U'}{\Omega_i} \gg \frac{k_z}{k_y} > 0$$

(the vorticity $\frac{dU}{dx}$ is the shear of the toroidal velocity). Relative to the ion gyrofrequency it is higher than the "angle" of propagation in the plane (radial, poloidal) or (y, z) . This can be the case if $k_z \rightarrow 0$, propagation almost perpendicular on the magnetic line.

We have followed the changes in the velocities of the ions.

Now we have to add the associated changes of the ion densities, via the continuity equation.

The change of the ion velocity is obtained by integrating $\delta u_z^{(2)} = -(u_x \delta t) \frac{dU}{dx}$

$$\begin{aligned} u_z^{(2)} &= - \int dt u_x \frac{dU}{dx} \\ &\approx - \frac{1}{\gamma} u_x \frac{dU}{dx} \end{aligned}$$

where the γ rate of growth is the order of magnitude of the time interval of interest. The equation of continuity is

$$\delta n^{(2)} = -N ik_z u_z^{(2)} \delta t$$

which can be formally integrated

$$\begin{aligned} n^{(2)} &= - \int dt N ik_z u_z^{(2)} \\ &\approx - \frac{1}{\gamma} N ik_z u_z^{(2)} \end{aligned}$$

To use the quasi-neutrality we consider the perturbation of the electron density in the wave field, taken as Boltzmannian

$$\delta n_e = N \frac{|e| \phi}{T_e}$$

(T_e in eV). Then

$$\begin{aligned} n^{(2)} &= \delta n_e \\ -\frac{1}{\gamma} N ik_z u_z^{(2)} &= N \frac{|e| \phi}{T_e} \\ -\frac{1}{\gamma} N ik_z \left(-\frac{1}{\gamma} u_x \frac{dU}{dx} \right) &= N \frac{|e| \phi}{T_e} \end{aligned}$$

and after replacing $u_x = \frac{-ik_y \phi}{B_0}$,

$$\frac{1}{\gamma^2} N ik_z \frac{dU}{dx} \left(\frac{-ik_y \phi}{B_0} \right) = N \frac{|e| \phi}{T_e}$$

or

$$\begin{aligned}
\gamma^2 &= \frac{1}{|e|} T_e k_z k_y \frac{1}{B_0} \frac{dU}{dx} \\
&= k_z k_y \frac{T_e}{m_i} \frac{1}{\frac{|e|B_0}{m_i}} \frac{dU}{dx} \\
&= k_z k_y \frac{c_s^2}{\Omega_i} U'
\end{aligned}$$

Regarding the change in the density produced by the gradient $\frac{dN}{dx}$.

Before, we have used the fact that the $E \times B$ drift produced by the wave on ions transports the ions along x , which are slower or faster than the ions that have been displaced at x_0 by the wave along \mathbf{B} .

Now we have to take into account that the advection on x also involves $\frac{dN}{dx}$.

Ions that are slower (smaller $U(x)$ at lower $x < x_0$) are also fewer since $N(x)$ decreases for $x < x_0$. The change of density by convection into x_0 is

$$\delta n^{(3)} = - (u_x \delta t) \frac{dN}{dx}$$

which we can formally integrate

$$\begin{aligned}
n^{(3)} &= - \int dt u_x \frac{dN}{dx} \\
&= - \frac{1}{\omega} u_x \frac{dN}{dx}
\end{aligned}$$

Applying again the neutrality with $\delta n_e = N \frac{|e|\phi}{T_e}$ and replacing u_x ,

$$\begin{aligned}
-\frac{1}{\omega} u_x \frac{dN}{dx} &= N \frac{|e|\phi}{T_e} \\
-\frac{1}{\omega} \left(\frac{-ik_y \phi}{B_0} \right) \frac{dN}{dx} &= N \frac{|e|\phi}{T_e}
\end{aligned}$$

Taking the absolute magnitude for ω

$$\begin{aligned}
|\omega| &= k_y \frac{T_e/m_i}{|e| B_0/m_i} \frac{1}{N} \frac{dN}{dx} \\
&= k_y \frac{c_s^2}{\Omega_i} \frac{d}{dx} \ln N = \omega_{dia}
\end{aligned}$$

This approximative value $|\omega| \sim \omega_{dia}$ allows a further analysis of the possibility of an KH instability.

In a time of the order $2\pi/\omega_{dia}$ the pattern of drift wave has travelled a full poloidal periodic unit, $2\pi/k_y$. The time for this travel must be compared with the growth rate. If the time for a significant increase of the wave perturbation,

γ^{-1} is longer than the time of wave propagation $2\pi/\omega_{dia}$ then the KH instability is inhibited since the accumulation of charge that is involved in the dynamics of the instability is depleted by the faster drift wave dynamics. Not enough time is left to the charges to accumulate and to pursue the rise of the perturbation since the drift wave has neutralized them by the "displacement" of the electric field \mathbf{E} wave profile. This provides the condition of stability

$$\gamma^2 \ll \omega_{dia}^2$$

$$\begin{aligned} \frac{U'}{\Omega_i} &< \frac{k_y T_e/m_i}{k_z \Omega_i^2} \frac{1}{L_n^2} \\ &= \frac{k_y c_s^2}{k_z \Omega_i^2} \frac{1}{L_n^2} = \frac{k_y}{k_z} \left(\frac{\rho_s}{L_n} \right)^2 \\ &\text{for stability} \end{aligned}$$

The paper uses the kinetic drift equation to study instabilities generated in a sheared parallel flow.

The equations of motion of the particle

$$\begin{aligned} \frac{d\mathbf{r}'}{dt} &= \mathbf{v}' \quad , \quad \text{with the initial condition } \mathbf{r}'(t' = t) = \mathbf{r} \\ \frac{d\mathbf{v}'}{dt} &= \frac{e}{m_i} \mathbf{v}' \times \mathbf{B}_0(x') \quad \text{with the condition } \mathbf{v}'(t' = t) = \mathbf{v} \end{aligned}$$

From these equations the following invariants result
energy

$$|\mathbf{v}'|^2 = |\mathbf{v}|^2 = v^2$$

and from canonical angular momentum conservation

$$x' + \frac{v_y'}{\Omega_i} = x + \frac{v_y}{\Omega}$$

The x direction is radial. The y direction is poloidal.

$$v_z' + \frac{x'}{L_s} v_y' + \frac{v_y'^2}{2\Omega_i L_s} = v_z + \frac{x}{L_s} v_y + \frac{v_y^2}{2\Omega_i L_s}$$

The shear length is L_s .

The distribution function

$$f_0 = \frac{n_0}{(2\pi v_{th,i}^2)^{3/2}} \exp \left\{ \frac{x + \frac{v_y}{\Omega_i}}{L_n} - \frac{1}{2v_{th,i}^2} \left[v^2 - 2 \left(v_z + \frac{x}{L_s} v_y + \frac{v_y^2}{2\Omega_i L_s} \right) \left[u + \frac{dU}{dx} \left(x + \frac{v_y}{\Omega_i} \right) \right] + \left[u + \frac{dU}{dx} \left(x + \frac{v_y}{\Omega_i} \right) \right]^2 \right] \right\}$$

It has been chosen

$$\begin{aligned} B_0 &> 0 \\ \frac{dU}{dx} &> 0 \end{aligned}$$

in the system $(\hat{\mathbf{e}}_z, \hat{\mathbf{e}}_x)$. The definition of the thermal velocity

$$v_{th,i} = \sqrt{\frac{k_{\text{Boltzmann}} T_i}{m_i}}$$

(remark no 2).

The velocities

$$\begin{aligned} u \text{ (of ions)} &\approx 0 \\ u \text{ (of electrons)} &= \text{finite, produces current} \end{aligned}$$

The sheared parallel velocity

$$\begin{aligned} \mathbf{u} &= \mathbf{U}(x) \\ &= \left(\frac{v_{th,i}^2}{\Omega_i L_n} \right) \hat{\mathbf{e}}_y + \left(u + \frac{dU}{dx} x \right) \left[\hat{\mathbf{e}}_z + \frac{x}{L_s} \hat{\mathbf{e}}_y \right] \end{aligned}$$

If gyroradius corrections are neglected

$$f_0 = \frac{n_0(x)}{(2\pi v_{th,i}^2)^{3/2}} \exp \left(- \frac{v_x^2 + v_y^2 + [v_z - (u + \frac{dU}{dx} x)]^2}{2v_{th,i}^2} \right)$$

Note the distinction between the need for the combination

$$x + \frac{v_y}{\Omega}$$

and the first correction to the distribution function

$$f = f_M - \frac{Iv_{\parallel}}{\Omega} \frac{\partial f_M}{\partial \psi} + g$$

The first is due to the Larmor gyration. It will involve in the problem the Bessel expansion of argument

$$\frac{k_{\perp} v_{\perp}}{\Omega}$$

and will modify the dispersion relations such that to depend on the gyration radius.

The second is a manifestation of the *neoclassical* correction that involves an equivalent *poloidal Larmor radius*, ρ_{θ} which comes from the drift of the particles.

End.

This work is actually for instabilities (see *instabilities*). However the solution is interesting here too

$$f(\mathbf{r}, \mathbf{v}, t) = \frac{e}{m_i} \int_{-\infty}^0 d\tau \nabla' \Phi(\mathbf{r}', t' = \tau + t) \cdot \nabla_{\mathbf{v}'} f_0(x', \mathbf{v}')$$

After applying the operator of derivation to the velocity ($\nabla_{\mathbf{v}'}$) one obtains from the Maxwellian a factor \mathbf{v}' . This gives

$$\begin{aligned} & \mathbf{v}' \cdot \nabla' \Phi(\mathbf{r}', \tau + t) \\ &= \frac{d\Phi}{dt} - \frac{\partial \Phi}{\partial \tau} \end{aligned}$$

The functions are expanded in

$$\exp(iky - i\omega t)$$

and it results

$$\begin{aligned} f(x, k, \mathbf{v}, \omega) &= \frac{e}{k_B T} f_0 \left\{ \phi(x, k, \omega) \right. \\ & \quad \left. + i \left[\omega - \omega_D - k_{\parallel} (u + U'x) \right. \right. \\ & \quad \left. \left. - \frac{k}{\Omega} U' (v_z - u - U'x) \right] \right\} \times I(x, k, \omega) \end{aligned}$$

Remember that k is poloidal.

The first line is the adiabatic response

$$\frac{e\Phi}{k_B T}$$

The second line is the basic resonance

$$\omega - k_{\parallel} v_{\parallel}$$

of the mode. Here v_{\parallel} is taken the *fluid* flow velocity.

The third line is the change that a flow produces in the wave resonance:

$$\frac{k_y}{\Omega} \sim \frac{1}{\text{velocity}}$$

$$U' \frac{k_y}{\Omega} v_z \sim \text{frequency}$$

The function

$$I(x, k_y, \omega)$$

$$= \int_{-\infty}^0 d\tau \Phi(x', k_y, \omega) \exp[-i\omega\tau + ik_y(y' - y)]$$

is the integral over the history along the trajectory $(x(\tau), y(\tau), z(\tau))$.

The "history" is

$$x' - x = \frac{v_{\perp}}{\Omega} [\sin(\Omega\tau - \zeta) + \sin\zeta]$$

$$y' - y = \frac{v_{\perp}}{\Omega} [\cos(\Omega\tau - \zeta) - \cos\zeta] + \frac{x}{L_s} v_z \tau$$

$$v_x = v_{\perp} \cos\zeta$$

$$v_y = v_{\perp} \sin\zeta$$

Now we have the "trajectory" of a generic particle and we must introduce it in the integrand of I .

The argument of Φ is *shifted spatial*

$$x' = x + \frac{v_{\perp}}{\Omega} [\sin(\Omega\tau - \zeta) + \sin\zeta]$$

where

$$\zeta \equiv \text{gyroangle}$$

In order to make the integration over τ we must *expand* formally the potential around the position x

$$\Phi(x') = \Phi(x) + \delta \frac{\partial \Phi}{\partial x} + \frac{1}{2} \delta^2 \frac{\partial^2 \Phi}{\partial x^2}$$

$$\delta \equiv \frac{v_{\perp}}{\Omega} [\sin(\Omega\tau - \zeta) + \sin\zeta]$$

Then we also replace $y' - y$ in the exponent.

One uses the expansion in Bessel functions, with

$$\exp[i\eta \cos \alpha] = \sum J_l(\eta) \exp\left[il\alpha + il\frac{\pi}{2}\right]$$

Now it is possible to carry out the τ integration (along the orbit)

$$\begin{aligned}
I &= \sum_{p=-\infty}^{p=+\infty} (-i)^p J_p \left(\frac{k_y v_\perp}{\Omega} \right) \exp(ip\zeta) \\
&\times \sum_{l=-\infty}^{l=+\infty} i^l \frac{\exp(-il\zeta)}{-i(\omega - k_\parallel v_z - l\Omega)} \left\{ J_l \left(\frac{k_y v_\perp}{\Omega} \right) \times \left[\Phi + \left(\frac{v_\perp}{\Omega} \sin \zeta - \frac{l}{k_y} \right) \frac{\partial \Phi}{\partial x} \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \left(\frac{l^2}{k_y^2} - 2 \frac{l}{k_y} \frac{v_\perp \sin \zeta}{\Omega} + \frac{v_\perp^2 \sin^2 \zeta}{\Omega^2} \right) \frac{\partial^2 \Phi}{\partial x^2} \right] \right. \\
&\quad \left. - \frac{1}{2} \frac{v_\perp}{k_y^2} \left[\frac{\partial}{\partial v_\perp} J_l \left(\frac{k_y v_\perp}{\Omega} \right) \right] \frac{\partial^2 \Phi}{\partial x^2} \right\}
\end{aligned}$$

This is the factor $I(x, k_y, \omega)$.

Now we must use the distribution function $f(x, k_y, \mathbf{v}, \omega)$ to calculate the *density* in the mode.

$$\int d^3 v f = \int v_\perp dv_\perp dv_z d\zeta f$$

The result

$$\begin{aligned}
e \int d^3 f &= -eN(x) \frac{1}{k_B T} \left(\Phi + \left\{ \frac{\omega - k_\parallel (u + U'x) - \omega_D}{\omega - k_\parallel (u + U'x)} \eta Z(\eta) - \frac{k_y U'}{k_\parallel \Omega} [1 + \eta Z(\eta)] \right\} \right. \\
&\quad \left. \times \left[\left(1 - \frac{k_y^2 v_{th}^2}{\Omega^2} \right) \Phi + \frac{v_{th}^2}{\Omega^2} \frac{\partial^2 \Phi}{\partial x^2} \right] \right)
\end{aligned}$$

where

$$\eta \equiv \frac{\omega - k_\parallel (u + U'x)}{|k_\parallel| v_{th}}$$

$$Z(\eta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dz \frac{\exp(-\frac{1}{2}z^2)}{z - \eta}$$

for $\text{Im} \eta > 0$

The Poisson equation

$$\begin{aligned}
&\rho_i^2 \frac{\partial^2 \Phi}{\partial x^2} \\
&+ (1 - k_y^2 \rho_i^2 \\
&\quad + \frac{(\tau + 1) + \frac{\omega - \omega_* - k_\parallel (u + U'x)}{|k_\parallel| v_{th,e}} Z \left(\frac{\omega}{|k_\parallel| v_{th,e}} \right)}{\left(\tau + \frac{\omega_*}{\omega} \right) \sigma Z(\sigma) - \frac{\tau k_y U'}{k_\parallel \Omega_i} [1 + \sigma Z(\sigma)]}) \Phi \\
&= 0
\end{aligned}$$

The notation

$$\begin{aligned}\sigma &\equiv \frac{\omega}{|k_{\parallel}| v_{th,i}} \\ \tau &\equiv \frac{T_e}{T_i} \\ \omega_* &= -k_y \frac{v_{th,e}^2}{\Omega_e} \frac{1}{L_n}\end{aligned}$$

The analysis of stability (solution of the Poisson equation) leads to the condition

$$\frac{dU}{dx} |L_n| \frac{1}{\sqrt{1 + \tau v_{th,i}}} > 1$$

for the excitation of the Kelvin Helmholtz instability ($\tau \equiv T_e/T_i$).

Stabilization by the magnetic shear (as is the case for drift waves) requires

$$L_s < 3 \frac{\sqrt{1 + \tau}}{\tau} |L_n|$$

If this magnetic shear cannot be realized then the suppression of the KH instability requires strong density gradient, *i.e.* very small $|L_n|$ such that

$$\frac{dU}{dx} |L_n| \frac{1}{\sqrt{1 + \tau v_{th,i}}} < 1$$

At this moment the KH instability is converted into a drift instability with very small growth rate.

This is the physical basis for the instability induced by the toroidal flow with sheared velocity.

The instability turns into a turbulent field when the amplitude is sufficiently high that the nonlinear coupling becomes significant.

It is this turbulence (associated and sustained by sheared toroidal flow) that produces a Reynolds stress which can transfer momentum to poloidal flow and so contribute to the transport barrier of the H mode.

One particular conclusion will be useful below.

The KH instability is suppressed if the gradient of the density becomes very steep (*i.e.* the length L_n becomes very small). This happens when the transport barrier (sheared poloidal rotation) is very efficient.

4 Poloidal/toroidal coupling through the Reynolds stress

Toroidal and poloidal rotation of the tokamak plasma are observed in all regimes. In **Su Yushmanov Dong Horton** it is recalled the relative magnitudes. The ratio of

1. the mean poloidal velocity

$$u_{\perp} = 0.44 v_{dia,e}$$

and

2. the mean toroidal velocity

$$u_{\parallel} = 3.5 v_{dia,e}$$

is about 1/9.

We will use expressions for the Reynolds stress derived from the turbulence induced by instabilities of sheared toroidal flow. [Su Yushmanov Dong Horton PoP 1 (1994) 1905].

The two components of the plasma rotation are defined such as to be function of the magnetic surface label, ψ . They are obtained first by projection on *parallel* $\hat{\mathbf{n}} = \mathbf{B}/B$ and on *perpendicular* direction $\hat{\mathbf{e}}_r \times \hat{\mathbf{n}}$, where the latter versors are obtained from $\nabla\psi/(2\pi RB_p)$ and from \mathbf{B}/B ,

$$u_{\parallel} = \frac{\langle \mathbf{B} \cdot \mathbf{v} \rangle}{B_0}$$

$$u_{\perp} = \frac{B_0}{2\pi R_0 B_{\theta}} \left\langle \left(\frac{\nabla\psi \times \mathbf{B}}{B^2} \right) \cdot \mathbf{v} \right\rangle$$

where

$$\mathbf{B} = R_0 B_{\varphi} \nabla\varphi + \frac{1}{2\pi} (\nabla\varphi \times \nabla\psi)$$

$$B_0 = \langle B^2 \rangle^{1/2} = \langle B_{\theta}^2 + B_T^2 \rangle^{1/2}$$

The average on the magnetic surface is

$$\langle A \rangle = \frac{\int A \frac{d\theta}{\mathbf{B} \cdot \nabla\theta}}{\int \frac{d\theta}{\mathbf{B} \cdot \nabla\theta}}$$

The **mean flow velocity** is

$$\mathbf{v} = \frac{\mathbf{B}}{B_0} \left(u_{\parallel} - \frac{B_T}{B_{\theta}} u_{\perp} \right) + u_{\perp} \frac{B_0}{R_0 B_{\theta}} R^2 \nabla\varphi$$

The first term is the incompressible plasma flow along the magnetic field lines.

The second term is the **rigid body rotation** of the plasma in the magnetic surface. This term does not contribute to the plasma viscosity. The plasma viscosity force is function of only the combination

$$u_{\parallel} - \frac{B_T}{B_{\theta}} u_{\perp}$$

The connection between the perpendicular plasma velocity \mathbf{u}_\perp and the **radial electric field** is given by

$$u_\perp = - \left(\frac{2\pi B_0}{R_0 B_\theta} \right) \left\langle \frac{R^2 B_\theta^2}{B^2} \right\rangle \left(\frac{d\Phi}{d\psi} + \frac{1}{en} \frac{dP}{d\psi} \right)$$

The velocity is defined such that positive E_r gives positive u_\perp which is the rotation in the ion diamagnetic direction.

The factor $2\pi R B_\theta$ comes from $\nabla\psi$ which is introduced to make possible the derivatives of Φ and p to ψ instead of r .

From momentum balance

$$nm (1 + 2\hat{q}^2) \frac{\partial u_\perp}{\partial t} = -F_\perp^R - F^R - F^{neo} - F_\perp^a$$

$$nm \frac{\partial u_\parallel}{\partial t} = \frac{B_\theta}{B_T} F^{neo} - F_\parallel^R - F_\parallel^a$$

where the forces with superscript R are due to the Reynolds stress in a turbulence.

The toroidal geometry is represented in the value of the quantity \hat{q}

$$2\hat{q}^2 = \left(\frac{B_T}{B_\theta} \right)^2 \left(1 - \frac{1}{\langle R^2 \rangle \langle R^{-2} \rangle} \right)$$

For large aspect ration, $\varepsilon \ll 1$,

$$2\hat{q}^2 \approx 2q^2$$

The velocity is taken as a sum of the mean velocity and a fluctuating velocity, $\tilde{\mathbf{v}}$ which will give the Reynolds stress.

In the expression of the mean velocity flow there are two terms. The first

$$\frac{\mathbf{B}}{B_0} \left(u_\parallel - \frac{B_T}{B_{pol}} u_\perp \right)$$

represents the *incompressible* plasma flow along the magnetic field lines.

The second

$$u_\perp \frac{B_0}{R_0 B_{pol}} R^2 \nabla \varphi$$

is a *rigid body rotation* within the magnetic surface. It has a direction $\hat{\mathbf{e}}_\varphi$ which is toroidal. This picture is also mentioned by **Hassam Kulsrud** who say that the motion of plasma consists of

1. the free motion along the magnetic flux tube
2. the motion of the flux tubes themselves

The second part of the velocity (the rigid body rotation) does not participate to the viscosity which finally will damp the poloidal rotation.

the viscosity will only depend on the combination exhibited in the first term

$$u_{\parallel} - \frac{B_T}{B_{pol}} u_{\perp}$$

The Reynolds stress produces terms that are marked by the upperscript R

$$\begin{aligned} F_{\parallel}^R &= \left\langle nm \frac{\mathbf{B}}{B_0} \cdot [(\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}}] \right\rangle \\ F_{\perp}^R &= \left\langle nm \frac{B_0 (\nabla \psi \times \mathbf{B})}{2\pi R_0 B_{pol} B^2} \cdot [(\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}}] \right\rangle \\ F_{\sim}^R &= \left\langle nm \frac{B_T}{B_{pol}} \left(\frac{B_0^2}{B^2} - 1 \right) \frac{\mathbf{B}}{B_0} \cdot [(\tilde{\mathbf{v}} \cdot \nabla) \tilde{\mathbf{v}}] \right\rangle \end{aligned}$$

The term of neoclassical damping effect on poloidal rotation is

$$F^{neo} = -\frac{B_T}{B_0 B_{\theta}} \left\langle B^2 (\mathbf{B} \cdot \nabla) \frac{P_{\parallel} - P_{\perp}}{2B^2} \right\rangle$$

is the viscosity of the plasma when it is pushed along nonuniform magnetic field (magnetic pumping effect).

The additional forces are related to the ripple or to the atomic processes. The two components are

$$\begin{aligned} F_{\perp}^a &= \left\langle \left[\frac{B_0}{2\pi R_0 B_{\theta}} \frac{\nabla \psi \times \mathbf{B}}{B^2} + \frac{B_T}{B_{\theta}} \left(\frac{B_0^2}{B^2} - 1 \right) \frac{\mathbf{B}}{B_0} \right] \cdot \mathbf{F}^a \right\rangle \\ F_{\parallel}^a &= \left\langle \frac{\mathbf{B}}{B_0} \cdot \mathbf{F}^a \right\rangle \end{aligned}$$

The neoclassical viscosity is

$$F^{neo} = -3\mu^{neo} \frac{B_T}{B_0^2} \left\langle \left(\frac{(\mathbf{B} \cdot \nabla) B}{B} \right)^2 \right\rangle \left(u_{\parallel} - \frac{B_T}{B_{\theta}} u_{\perp} - k_{\nu_*} \frac{1}{e B_{\theta}} \frac{dT_i}{dr} \right)$$

where the neoclassical viscosity coefficient is (for velocities which are much less than the sound velocity)

$$\begin{aligned} \mu^{neo} &\approx R_0 q \frac{nm v_{th} \nu_*}{1 + \nu_*} \frac{1}{1 + \varepsilon^{3/2} \nu_*} \\ \nu_* &= \nu \varepsilon^{3/2} \frac{qR}{v_{th}} \end{aligned}$$

and ν is the ion collision frequency.

The coefficient k_{ν_*} describes the relative effect of the **parallel heat flux** on the longitudinal viscosity (**Hazeline**)

$$\begin{aligned} k_{\nu_*} &= 1.17 & \text{for } \nu_* &\ll 1 \\ k_{\nu_*} &= -0.5 & \text{for } 1 &\ll \nu_* \ll \varepsilon^{-3/2} \\ k_{\nu_*} &= -2.1 & \text{for } \nu_* &\gg \varepsilon^{-3/2} \end{aligned}$$

We **NOTE** that the first part of the velocity paranthesis

$$u_{\parallel} - \frac{B_T}{B_{\theta}} u_{\perp}$$

can be written

$$-\frac{B_T}{B_p} \left(u_{\perp} - \frac{B_p}{B_T} u_{\parallel} \right)$$

and we see that the last term $B_p u_{\parallel} / B_T$ is the projection of the parallel velocity on the *poloidal direction*. Then the paranthesis is a strange difference

$$u_{\perp} - u_{\theta}^{\parallel}$$

END

the velocity that is involved in the *standard viscosity* is the combination

$$u_{\parallel} - \frac{B_T}{B_{\theta}} u_{\perp}$$

The parallel velocity is

$$\mathbf{v}_{\parallel} = \frac{\mathbf{B}}{B_0} \left(u_{\parallel} - \frac{B_T}{B_{\theta}} u_{\perp} \right)$$

and the *rigid body rotation of the magnetic surface* is

$$u_{\perp} \frac{B_0}{R_0 B_p} R^2 \nabla \varphi$$

The equilibrium value of the poloidal rotation

$$\begin{aligned} u_{\theta} &= -u^{neo} \\ &= -k_{\nu_*} \frac{1}{e B_T} \frac{dT_i}{dr} \left(\frac{L_n}{\rho_s c_s} \right) \end{aligned}$$

The equilibrium poloidal velocity is determined by the ion temperature gradient and it is

$$\begin{aligned} &\text{ion diamagnetic direction} && \text{for } \nu_* > 1, \quad k_{\nu_*} < 0 \\ &\text{electron diamagnetic direction} && \text{for } \nu_* < 1, \quad k_{\nu_*} > 0 \end{aligned}$$

There are forces due to the nonambipolar processes (like: charge exchange, ion-direct-loss, neutral beams). They are represented in a simplified form:

$$F_{\perp}^a = nm\nu_{\perp}^a (u_{\perp} - u_{\perp}^a)$$

$$F_{\parallel}^a = nm\nu_{\parallel}^a (u_{\parallel} - u_{\parallel}^a)$$

The resulting equations governing the plasma **perpendicular** and **parallel** motions are

$$(1 + 2q^2) \frac{\partial u_{\perp}}{\partial t} = -\nu^{nc} (u_{\perp} - \Theta u_{\parallel} + u^{nc}) - \\ -\nu_{\perp}^a (u_{\perp} - u_{\perp}^a) - \\ -\frac{\partial}{\partial x} \langle \tilde{v}_x \tilde{v}_{\perp} \rangle - \\ -2q \left\langle \cos \theta \frac{\partial}{\partial x} (\tilde{v}_x \tilde{v}_{\parallel}) \right\rangle$$

$$\frac{\partial u_{\parallel}}{\partial t} = -\nu^{nc} \Theta (-u_{\perp} + \Theta u_{\parallel} - u^{nc}) - \\ -\nu_{\parallel}^a (u_{\parallel} - u_{\parallel}^a) - \\ -\frac{\partial}{\partial x} \langle \tilde{v}_x \tilde{v}_{\parallel} \rangle$$

where

$$\Theta = \frac{r}{qR} = \frac{B_{\theta}}{B_{\varphi}} \ll 1$$

$$u^{nc} = k_{\nu_*} \frac{1}{eB_{\varphi}} \frac{dT_i}{dr} \left(\frac{L_n}{\rho_s c_s} \right)$$

is the equilibrium velocity of the **poloidal velocity**, (**Hazeltine**) which is

$$u_{\theta} = u_{\perp} - \Theta u_{\parallel}$$

The **toroidal velocity** is

$$u_{\varphi} = u_{\parallel} + \Theta u_{\perp}$$

Citing **Su, Yushmanov**,

$$\nu^{nc} = \frac{3\mu^{nc} B_T^2}{nm B_{\theta}^2 B_0^2} \frac{1}{B} \left\langle \left(\frac{(\mathbf{B} \cdot \nabla) B}{B} \right)^2 \right\rangle \\ \approx \frac{3 B_T^2}{2 B_{\theta}^2} \frac{\sqrt{\varepsilon}}{1 + \nu_*} \left(\frac{\nu L_n}{c_s} \right)$$

$$S_0 = \frac{B_\theta}{B_T}$$

$$u^{nc} = k_{\nu^*} \frac{c}{eB_T} \frac{dT_i}{dr} \left(\frac{L_n}{\rho_s c_s} \right)$$

(Hazeltine)

The differential operator

$$\nabla_{\parallel} = \frac{\partial}{\partial z} + \frac{L_n}{\rho_s} S_0 \frac{\partial}{\partial y}$$

it takes into account the magnetic shear (the deviation of the direction of the magnetic field line relative to z).

$$\nabla_{\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

A useful approximation

$$\frac{\partial}{\partial \psi} = \frac{1}{2\pi R B_\theta} \frac{\partial}{\partial x}$$

for $\varepsilon \ll 1$.

In usual situations there is damping of the poloidal velocity

$$u_\theta = u_{\perp} - S_0 u_{\parallel}$$

which is reduced to the neoclassical equilibrium (**Hazeltine**)

$$u_\theta^{neo} = \left(\frac{L_n}{\rho_s c_s} \right) \times k_{\nu^*} \frac{1}{eB_T} \frac{dT}{dr}$$

Therefore there are two opposite actions that compete to determine the poloidal flow.

- the neoclassical sources of damping

$$\nu^{neo} , \nu_{\perp}^a , \nu_{\parallel}^a$$

and

- the turbulent Reynolds stress

$$\pi_{x\perp} = \langle \tilde{v}_x \tilde{v}_{\perp} \rangle$$

$$\pi_{x\parallel} = \langle \tilde{v}_x \tilde{v}_{\parallel} \rangle$$

We now have admitted the presence of equilibrium (non-turbulent) flows, u_{\perp} and u_{\parallel} .

On this ground we consider turbulent fluctuations

$$\tilde{\phi} \quad \text{and} \quad \tilde{v}_{\parallel}$$

The equations for them will reflect their dynamics sitting on the equilibrium evolutions of the basic flows AND the effect of the turbulence as transfer of momentum.

From the dynamical equations we derive equations for the fluctuations

$$\begin{aligned} & (1 - \nabla_{\perp}^2) \frac{\partial \tilde{\phi}}{\partial t} + v_{dia} \frac{\partial \tilde{\phi}}{\partial y} \\ & + \left[\frac{\partial^2 u_{\perp}}{\partial x^2} + (u_{\perp} + S_0 u_{\parallel}) (1 - \nabla_{\perp}^2) \right] \frac{\partial \tilde{\phi}}{\partial y} \\ & - \left[(-\nabla_{\perp} \tilde{\phi} \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] \nabla_{\perp}^2 \tilde{\phi} \\ = & -\nabla_{\parallel} \tilde{v}_{\parallel} \\ & -\mu_{\perp} \nabla_{\perp}^4 \tilde{\phi} \\ & \frac{\partial \tilde{v}_{\parallel}}{\partial t} + (u_{\perp} + S_0 u_{\parallel}) \frac{\partial \tilde{v}_{\parallel}}{\partial y} - \frac{\partial u_{\parallel}}{\partial x} \frac{\partial \tilde{v}_{\parallel}}{\partial y} \\ & + \left[(-\nabla_{\perp} \tilde{\phi} \times \hat{\mathbf{e}}_z) \cdot \nabla_{\perp} \right] \tilde{v}_{\parallel} \\ = & -\nabla_{\parallel} \tilde{\phi} \\ & +\mu_{\perp} \nabla_{\perp}^2 \tilde{v}_{\parallel} \end{aligned}$$

The basic flows

$$u_{\perp} \quad \text{and} \quad u_{\parallel}$$

are solutions of the previous system.

The normalizations

$$\begin{aligned} (x', y') &= (x, y) \frac{1}{\rho_s} \\ z' &= z \frac{1}{L_n} \\ t' &= t \frac{c_s}{L_n} \\ \phi' &= \left(\frac{L_n}{\rho_s} \right) \frac{e\phi}{T_e} \end{aligned}$$

$$(u_{\perp}, u_{\parallel}) \quad \text{and} \quad \left(\tilde{v}_{\parallel}, \tilde{\mathbf{v}}_{\perp} = \left(\frac{\rho_s c_s}{L_n} \right) \hat{\mathbf{n}} \times \nabla_{\perp} \tilde{\phi} \right)$$

are normalized to

$$v_{dia,e} = \frac{\rho_s c_s}{L_n}$$

The energy coupling through Reynolds stress involves the following integral quantities

$$E_{bg,1} = \frac{1}{2} \int \frac{dV}{L_x L_y} (1 + 2q^2) u_{\perp}^2$$

$$E_{bg,2} = \frac{1}{2} \int \frac{dV}{L_x L_y} u_{\parallel}^2$$

and the fluctuations

$$E_{fl,1} = \frac{1}{2} \int \frac{dV}{L_x L_y} \left[\tilde{\phi}^2 + \left(\nabla_{\perp} \tilde{\phi} \right)^2 \right]$$

$$E_{fl,2} = \frac{1}{2} \int \frac{dV}{L_x L_y} \tilde{v}_{\parallel}^2$$

the stress tensors

$$\pi_{x\perp} = \langle \tilde{v}_x \tilde{v}_{\perp} \rangle = - \int \frac{dy}{L_y} \frac{\partial \tilde{\phi}}{\partial x} \frac{\partial \tilde{\phi}}{\partial y}$$

$$\pi_{x\parallel} = \langle \tilde{v}_x \tilde{v}_{\parallel} \rangle = - \int \frac{dy}{L_y} \frac{\partial \tilde{\phi}}{\partial y} \tilde{v}_{\parallel}$$

This two sets of equations (one for large scale, "background" flows and one for turbulent fluctuations $\tilde{\phi}$ and \tilde{v}_{\parallel}) must be solved numerically.

We refer to this system for the case where there is input of toroidal momentum from the NBI. In the basic state (without the NBI) the shear of the toroidal rotation is not sufficient to generate turbulence to a level that can transfer momentum to poloidal rotation. With NBI the shear u'_{\parallel} is sufficient to destabilise KH mode and rise the turbulence to the level that allows Reynolds stress transfer to the poloidal rotation. The stress is

$$\pi_{x\perp}$$

5 Evolution of the poloidal velocity

A system of equations for three quantities is written (Hassam IAEA) to derive the evolution of the poloidal velocity:

1. averaged density $\langle n \rangle$

2. the **toroidal angular momentum** $\langle nRv_\varphi \rangle$ and
3. the **circulation** $\langle v_\parallel B \rangle$

In the absence of sources the density and the toroidal angular momentum are **conserved**. The *circulation* is only **convected**. The following system is obtained, the first two eqs. representing the two conservation laws:

$$\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rn\bar{v}_r) = 0$$

$$\frac{\partial}{\partial t} (nV_\varphi) + \frac{1}{r} \frac{\partial}{\partial r} [rn (V_\varphi \bar{v}_r - qV_\theta \tilde{v}_r)] = 0$$

$$\frac{\partial}{\partial t} [V_\varphi + \Theta (1 + 2q^2) V_\theta] + \bar{v}_r \frac{\partial V_\varphi}{\partial r} - \tilde{v}_r \frac{\partial}{\partial r} (qV_\theta) + \text{magnetic pumping} = 0$$

where

$$\langle n \rangle = n(r)$$

$$nV_\varphi(r) \equiv \frac{1}{R_0} \langle nv_\varphi R \rangle$$

$$V_\theta(r) \equiv \frac{1}{R_0} \langle v_\theta R \rangle$$

with the **magnetic field**

$$\mathbf{B} \equiv [0, \Theta(r), 1] \frac{B_0(r) R_0}{R}$$

$$R = R_0 + r \cos \theta$$

$$\Theta(r) = \frac{r}{qR_0} = \frac{B_\theta}{B_T} \ll 1$$

We **note** that the new "velocity" variables are defined such that they are only functions of ψ (or r). The *surface averaging* operation is

$$\langle f \rangle \equiv \oint \frac{d\theta}{2\pi} (1 + \varepsilon \cos \theta) f$$

which should be compared with

$$\langle f \rangle = \frac{\oint \frac{d\theta}{\mathbf{B} \cdot \nabla_\theta} f}{\oint \frac{d\theta}{\mathbf{B} \cdot \nabla_\theta}}$$

$$\frac{d\theta}{\mathbf{B} \cdot \nabla_\theta} = \frac{1}{B} \frac{d\theta}{\nabla_\parallel \theta} = \frac{1}{B} qR d\theta = \frac{1}{B} \frac{rB_T}{RB_\theta} R d\theta$$

The quantities \bar{v}_r and \tilde{v}_r represent **radial diffusion velocities** arising from *electron-ion momentum transfer*. The force associated to this momentum transfer is

$$\mathbf{R}_\perp \text{ with the direction of } \mathbf{B} \times \nabla n$$

which is \perp , i.e. approximately poloidal. When there is a force that is *in surface* and is perpendicular, there is a resulting radial diffusion flux

$$nv_r = \frac{R_\perp}{eB}$$

from which the two definitions are obtained:

$$\begin{aligned}\bar{v}_r &\equiv \langle v_r \rangle \\ \tilde{v}_r &\equiv \langle 2 \cos \theta v_r \rangle\end{aligned}$$

The resulting **equation for the plasma poloidal velocity** is

$$\begin{aligned}\Theta (1 + 2q^2) \left(\frac{\partial V_\theta}{\partial t} + \gamma_{MP} V_\theta \right) + qV_\theta \frac{1}{nr} \frac{\partial}{\partial r} (nr\tilde{v}_r) &= 0 \\ \gamma_{MP} &\simeq \frac{3}{4} \left(1 + \frac{1}{2q^2} \right)^{-1} \left(\frac{l}{qR} \right)^2 \nu_{ii}\end{aligned}\tag{1}$$

where l is the mean-free path.

6 Conclusion

The sheared toroidal rotation in tokamak is observed in many experiments. In general it is assumed that the plasma rotation is the reason for the decrease of the transport rate and for the access to high confinement. However the toroidal rotation can only act along the parallel connection length of an instability which is very large, $\sim qR$. The efficiency of the effect against the instabilities (by the flow-change of k_{\parallel}) is small. On the contrary, the perpendicular to \mathbf{B} velocity has effect on the instabilities in two ways:

(1) affecting the linear stage by shifting the resonant surface and reducing the rate of extraction of the free energy

(2) by acting directly on the convective cells of the unstable modes (eddies) by tearing apart them and reducing their (transversal) dimension. Since this spatial parameter of the instability enters to the diffusion coefficient to power two, the reduction of the transport is substantial.

The poloidal sheared plasma rotation is the basis for reaching the H mode.

We then have (1) an ubiquitous sheared toroidal rotation with low efficiency in controlling transport but with higher possibility of producing turbulence; we have shown, the Kelvin-Helmholtz turbulence; and we have (2) a poloidal rotation, with high efficiency in reducing transport but which must confront the plasma poloidal inertia and the strong mechanism of damping due to magnetic pumping; then the poloidal rotation needs extrinsic factors to be sustained.

We use the fact that the turbulence generated by the sheared toroidal rotation (KH) can transfer momentum to the poloidal rotation through Reynolds stress. This seems to explain the fact, observed in experiments, that, when it

takes place the transition from L to H there is a substantial sudden reduction in the toroidal rotation and simultaneous an increase of the poloidal rotation. Since the transition is frequently supported by NBI one can say that the sheared toroidal rotation has transferred momentum to the poloidal rotation. In turn, the so-sustained sheared poloidal rotation is able to act against the transport coming from the plasma core by creating a transport barrier.

The transport barrier help increasing the gradients of the main parameters (n , T) (which is actually the formation of the pedestal). Rising the gradients has an effect of reducing the linear growth rate of the KH instability, *i.e.* of the turbulence generated by the sheared toroidal rotation. The decay of this turbulence reduces the Reynolds stress which is the "source" of poloidal rotation. Then the poloidal rotation is damped and the transport barrier is reduced or suppressed.

We claim that this is a plausible scenario for the interplay toroidal / poloidal and provides an explanation for the random bursts of loss at the plasma edge (even in absence of the ELMs).

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