MST1 Complementar

Running scale separation between turbulence and structures and transient enhancement of transport

1 Introduction

Plasma instabilities are sustained by extraction of free energy from the background gradients, characterized the linear growth rate γ. However this is not the unique source of spectral energy.

The linear dispersion relation provides a characterization of the conditions (plasma parameters) that are favorable to growth or damping of a particular eigenmode. However an initialization consisting of high amplitude pulse may place the system (plasma) far beyond linear considerations and able to generate robust and resilient structures, like solitons. They are not accessible to the linear prediction.

We consider that in a finite spatial region of the system (the region of interest for the transient convection cells) arrive pulses generated by random processes that take place in distant regions, *e.g.* in plasma core. These can be saw-teeth, transient magnetic islands, etc. These fluctuations act as a perturbation on the dynamics of linear extraction of free energy, spectral coupling and saturation (via dissipation). There is no connection between a perturbation with a function $\Phi^{pert}(\mathbf{x},t)$, born somewhere in the system and propagating to the edge, - and the dispersion relation of the eigenmodes which are local to the region of our interest. The spectral content of the incoming perturbation $\Phi^{pert}(\mathbf{x},t)$ has no connection with the linear dispersion relation. This means that the pulse will deposit some energy on intervals of the spectrum in a way that is not correlated with the growth of damping according to the linear dispersion relation. The random perturbation consisting of pulses creates condition for the generation of small scale structures that would only rarely be spontaneously created in the equilibrium (based on extraction of free energy) spectrum of turbulence.

Since the amplitude of the pulses can be high, it becomes possible to generate strongly nonlinear structures. In the drift waves, such structures are monopolar or dipolar vortices.

The process of coalescence of vortices consists in voiding of energy the region of small spatial scales and transfering this energy to large scales. The vortex interaction, attraction, merging, leading to transfer of energy from the small spatial scales to large scales (convective cells) is a compact event of reorganization similar to an avalanche, but in $\mathbf{k}$ space.

The random perturbations traversing the region of interest (and independent on the linear extraction of energy from gradients) are dropping energy to random $\mathbf{k}$-intervals on the spectrum. A model that is adequate to describe this is the Kardar Parisi Zhang equation in $\mathbf{k}$ space. For the avalanches in $\mathbf{k}$ space the analytical description of Hwa and Kardar can be taken as reference. Let us enumerate the primary elements of our approach.

- A pulse that acts as a perturbation coming from "external" domain (most probable from the plasma core) will produce vortices.
- the vortices are nonlinear structures with high resiliancy (compared with the perturbations that are added to some spectral interval where the linear dispersion has found damping). The vortices are similar to solitons and they keep the energy of the initial perturbation for a time that is longer than $\sim |\gamma|^{-1}$.
- this creates *metastable* states
- the vortices have the tendency to propagate in the poloidal direction, like the ion diamagnetic flow v_{dia};
- The vortices that propagate with velocity that is smaller than the diamagnetic one lose energy via radiation of drift waves. They have the tendency of being localized on r. The profile tends to be smeared out along y (poloidal) while radially they remain localised for longer time
- There is attraction, merging and generation of larger spatial convection flows. In the spectrum the small scales are voided of energy and the large scales are charged with the energy released from the small scales. There is no particular spectral transport of the energy, as a cascade. It simply is a reconfiguration of the flow (merging of vortices) which costs low energy, only necessary for reconnection of the patterns of flow and generation of a convective cell.
- This may be seen as avalanches in $\mathbf{k}$, a mechanism of voiding the energy from small scales.

2 Spectral dynamics and cuasi-coherent vortices

We are interested in the dynamics in the spectral space $(\mathbf{k}, \omega)$ for a turbulence in tokamak generated by drift waves. Usually the modulation of the amplitude of small scale oscillations is examined on multiple space-time scales. For high amplitude strongly nonlinear structures that undergo coalescence this method is not useful.

We investigate an intermediate level of the manifestation of the turbulent field. This intermediate level consists of vortices (as cuasi-coherent structures) with transient life.

For the sudden reorganization of the spectral energy distribution the Kardar Parisi Zhang model may be useful. The amplitude of energy in $\mathbf{k}$ space is like an interface $h(\mathbf{k},t)$ that grows by addition of random deposition.

$$\frac{\partial h}{\partial t} = \frac{1}{2}\lambda\left(\boldsymbol{\nabla} h\right)^2 + \nu \boldsymbol{\nabla}^2 h + \eta\left(\mathbf{k},t\right)$$

The connection resides in the fluxes $\sim \boldsymbol{\nabla} h$ that simulate the loss of action quanta N_k toward the neighbor intervals in the spectrum. This corresponds to the process which is discussed here: the formation of vortices is followed by coalescence and this leads to spatial extension of the nonlinear flow pattern. Equivalently, the spectral energy will not be located at the small scales of the vortex but at the large spatial scales of convection cells. This is a transition that does not involve cascade. The processes of vortex merging are similar to *avalanches* in $\mathbf{k}$ space, since they void of energy some spectral intervals to enhance others (large scale). We however maintain the diffusion $\nu\boldsymbol{\nabla}^2 h$ as in Wave Kinetic Equation.

We want to investigate the formation of large scale convective structures (convective cells) from the coalescence of vortices.

2.1 The equations leading to vortex structures

The ion equations

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\right)\mathbf{v} = -\frac{1}{\Omega_{ci}}\widehat{\mathbf{n}}\times\mathbf{v} - \frac{|e|}{m_i}\boldsymbol{\nabla}\phi$$

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}\cdot(n\mathbf{v}) = 0$$

The electrons are assumed to move freely along the magnetic line, $\mathbf{B} = B\widehat{\mathbf{n}}$. Dissipation is neglected.

The velocity and vorticity $\mathbf{v} = \frac{-\boldsymbol{\nabla}\Phi\times\widehat{\mathbf{n}}}{B}$ and $\boldsymbol{\omega} = \boldsymbol{\nabla}\times\mathbf{v} = \Omega_{ci}\frac{\rho_s^2}{T_e}\boldsymbol{\nabla}_\perp^2\Phi$ where $\Omega_{ci} = \frac{|e|B}{m_i}$, $\rho_s = \frac{c_s}{\Omega_{ci}}$, $c_s = \sqrt{\frac{T_e}{m_i}}$. There is a gradient of the density $L_n^{-1} = -\frac{d\ln n_0}{dx}$ and it is defined $\varepsilon_n = \frac{\rho_{s0}}{L_n}$ with the parameters calculated at a reference (constant) electron temperature T_0. The potential is normalized

$$\phi = \frac{L_n}{\rho_{s0}}\frac{e\Phi}{T_0}$$

and also the temperature $T(x) = T_e(x)/T_0$, x being the radial coordinate and y the poloidal one.

The equation for the ion fluid

$$\frac{\partial}{\partial t}\left(\frac{1+\varepsilon_n\boldsymbol{\nabla}_\perp^2\phi}{n}\right) + \left[\left(-\boldsymbol{\nabla}_\perp\phi\times\widehat{\mathbf{n}}\right)\cdot\boldsymbol{\nabla}\right]\left(\frac{1+\varepsilon_n\boldsymbol{\nabla}_\perp^2}{n}\right) = 0$$

This corresponds to the Ertel's theorem that the potential vorticity $(\omega + \Omega_i)/n$ is a Lagrangian invariant.

It is assumed that the electrons are sufficiently fast to take a Boltzmann distribution

$$n = n_0(x) \exp\left(\frac{|e|\,\Phi}{T_e(x)}\right) = n_0(x) \exp\left(\varepsilon_n \frac{\phi}{T(x)}\right)$$

2.2 The vortex

Consider a structure that travels in the poloidal direction

$$\phi = \phi(x, y - ut)$$

Then the equation becomes

$$\begin{aligned} -u\frac{\partial G}{\partial y} + \left[\left(-\boldsymbol{\nabla}_\perp \phi \times \widehat{\mathbf{n}}\right) \cdot \boldsymbol{\nabla}_\perp\right] G &= 0 \\ \text{where } G &\equiv \frac{1 + \varepsilon_n \boldsymbol{\nabla}_\perp^2 \phi}{n} \end{aligned}$$

The equation is solved by

$$G = \frac{1 + \varepsilon_n \boldsymbol{\nabla}_\perp^2 \phi}{n_0(x) \exp\left(\varepsilon_n \frac{\phi}{T(x)}\right)} = F(\phi - ux)$$

for an arbitrary function F. It is chosen a function F as

$$F(X) = \frac{1}{n_0\left[-\frac{X}{u}\right]}$$

where

$$X = \phi - ux$$

In addition it is adopted the simple exponential profile for the equilibrium density

$$n_0(x) = \exp(-\varepsilon_n x)$$

This means

$$\begin{aligned} F(X) &= \exp\left[-\varepsilon_n \frac{X}{u}\right] \\ F(\phi - ux) &= \exp\left(-\varepsilon_n \frac{\phi - ux}{u}\right) \end{aligned}$$

and

$$\frac{1 + \varepsilon_n \boldsymbol{\nabla}_\perp^2 \phi}{n_0(x) \exp\left(\varepsilon_n \frac{\phi}{T(x)}\right)} = \exp\left(-\varepsilon_n \frac{\phi - ux}{u}\right)$$

or

$$\begin{aligned} 1+\varepsilon_n\phi &= \exp\left(-\varepsilon_n x\right)\exp\left(\varepsilon_n\frac{\phi}{T\left(x\right)}\right)\exp\left(-\varepsilon_n\frac{\phi-ux}{u}\right) \\ &= \exp\left[-\varepsilon_n x+\varepsilon_n\frac{\phi}{T\left(x\right)}-\varepsilon_n\frac{\phi}{u}+\varepsilon_n x\right] \\ &= \exp\left[\varepsilon_n\left(\frac{1}{T\left(x\right)}-\frac{1}{u}\right)\phi\right] \end{aligned}$$

The diamagnetic velocity is

$$V_{dia}=\frac{1}{n}\frac{1}{m_i\Omega_{ci}}\frac{dp_i}{dr}=\frac{c_s^2}{\Omega_{ci}}\frac{1}{L_n}=c_s\frac{\rho_s}{L_n}=\varepsilon_n c_s$$

Then, V_{dia} is normalized to c_s the sound speed

$$V_{dia}/c_s\rightarrow v_{dia}$$

Expanding the diamagnetic velocity

$$\boldsymbol{\nabla}_\perp^2\phi=k_0^2\phi+\left(\frac{1}{2}\frac{dv_{dia,0}}{dx}\frac{1}{u^2}\right)\phi^2$$

where

$$k^2\left(u,x\right)=\frac{1}{T\left(x\right)}-\frac{v_{dia}\left(x\right)}{u}$$

is retained in only zeroth order

$$k^2\left(u,x\right)=k_0^2+\alpha x+...$$

$$\alpha=\kappa_T-\frac{dv_{dia}}{dx}\frac{1}{u}$$

$$\kappa_T=\rho_{s0}\frac{d}{dx}\left(\frac{1}{T}\right)$$

$$k_0^2=1-\frac{v_{dia,0}}{u}$$

and k_0^2 is a small quantity.

When

$$k^2=k_0^2+\alpha x+...<0$$

there is radiation emitted by the vortex and it decays.

The approximate solution (**Petviashvili**)

$$\begin{aligned} \phi\left(x,y,t\right) &\approx -2.4k_0^2\frac{2u^2}{\left(\frac{dv_{dia,0}}{dx}\right)} \\ &\quad\times\left\{\cosh\left[\frac{3}{4}k_0\sqrt{x^2+\left(y-ut\right)^2}\right]\right\}^{-4/3} \end{aligned}$$

There is a connection between the amplitude and the velocity of propagation of the vortex (**Horton Meiss**)

$$u = \frac{1}{2}\left(v_{dia,0} + \sqrt{v_{dia,0}^2 + 0.83\left|\frac{\partial v_{dia,0}}{\partial x}\phi_m\right|}\right)$$

where

$$\phi_m = \max\left|\phi\left(x,y,t\right)\right|$$

This is the relationship between the ampitude ϕ_m and the speed u. We will use this relation to calculate the number of vortices that can be generated from an initial pulse-perturbation.

2.3 The number of vortices that result from a perturbation

Digression

For reference we review the method of calculation of the number of solitons that can result from an initial perturbation. The discussion below follows **Horton Meiss** and it refers to solitons of the $1D$ nonlinear equation KdV.

Horton and Meiss discuss the KdV equation for the propagation of a structure along the poloidal direction.

The reason to focus on the KdV is a transformation of the polarization term in the $1D$ version of the CHM equation into a higher order derivative

$$\frac{\partial}{\partial t}\left(\frac{\partial^2\varphi}{\partial y^2}\right) \rightarrow \frac{\partial^3\varphi}{\partial y^3}$$

and with this transformation the equation becomes KdV. This term occurs with the coefficient β. The transformation

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial y}$$

means that the soliton moves in the y direction with constant velocity.

A KdV soliton is

$$\varphi\left(y,t|y_0,u\right) = -3\left(u-1\right)\sec h^2\left[\frac{1}{2}\sqrt{u-1}\left(y-y_0-ut\right)\right]$$

The equation is KdV, it is $1D$ along the poloidal direction y on the circumference $L = 2\pi r$. The scenario known in the case of KdV consists of the transformation of an initial pulse into a number of solitons plus an oscillating tail.

We consider that this physical case is close to our problem of vortices generated by an initial perturbation.

The work of Horton Meiss utilizes the results of **Karpman Sokolov**.

The paper **Karpman Sokolov** where the number of solitons from an initial pulse is calculated, finds a scaling for this number with the dimensionless *similarity parameter*

$$\sigma = l\sqrt{\frac{u_0}{\beta}}$$

Here β is the coefficient of the term

$$\beta\frac{\partial^3 u}{\partial y^3}$$

with $[\beta] = (\text{length})^3 / (\text{time})$ then $\sigma = (\text{length})\sqrt{\frac{\text{length}}{\text{time}}\frac{1}{\frac{(\text{length})^3}{\text{time}}}}$ =dimensionless.

$$\begin{aligned} u_0 &\equiv \text{characteristic amplitude} \\ &\quad \text{of the initial perturbation} \end{aligned}$$

$$\begin{aligned} l &\equiv \text{characteristic width of the} \\ &\quad \text{initial perturbation} \end{aligned}$$

The scaling of **Karpman Sokolov** is expressed in terms of the *similarity parameter* σ

$$N \sim \frac{1}{\sqrt{3}\pi}\sigma + ...$$

According to **Horton Meiss**, the work of **Karpman and Sokolov** finds the number of solitons

$$\begin{aligned} n_u[\varphi] &= \frac{\sqrt{3}}{4\pi}\int_{\varphi<-\frac{A}{2}} \frac{dy}{\sqrt{-2\varphi(y) - A}} \\ A &= 3(u-1) \text{ it is } -(\text{amplitude}) \end{aligned}$$

see the expression of the soliton solution

$$u > v_{dia} \quad \equiv \text{speed of soliton displacement}$$

The initial state is random as a white noise. In every point (x_i, y_i) where $i = 1, ..., n$ of a discretization of the region where the external pulse has produced vortices, the amplitude of the fluctuation is random

$$\varphi(x_i, y_i)$$

with Gaussian probability. In that point the amplitude $\varphi(x_i, y_i)$ has produced $n[\varphi]$ solitons. This must be averaged over the random spreading of the amplitude of the initial perturbation, as explained. This is

$$f_s(u) = \frac{1}{Z}\prod_{i=1}^{n}\int_{-\infty}^{\infty} d\varphi_i \ \exp\left[-\frac{(\varphi(x_i, y_i))^2}{2\varphi_0^2}\right] \ n_u[\varphi]$$

After calculating this integral (with the expression of $n_u[\varphi]$ from the work of **Karpman Sokolov**, see above) one can obtain the total number, by integrating over all spees of propagation, u, that are bounded from below by the diamagnetic velocity

$$\int_{v_{dia,0}}^{\infty} du\ f_s(u) = N_{solit}$$

Then the average number of solitons that can be generated from random initial perturbation is (**Horton Meiss**)

$$\begin{aligned} N_{solit} &= \alpha L\sqrt{\varphi_0} \\ \alpha &= \frac{\Gamma\left(\frac{3}{4}\right)}{\sqrt{12\sqrt{2}\pi^3}} \end{aligned}$$

where φ_0 is the average amplitude of the ensemble of random initial fluctuations

$$\begin{aligned} \left\langle\varphi^2\right\rangle &= \varphi_0^2 \\ \text{or}\ \ \varphi_0 &= \sqrt{\left\langle\left(\frac{e\Phi}{T_e}\right)^2\right\rangle} \end{aligned}$$

If we just use the result for $1D$ solitons to our case of $2D$ vortices, we would have

$$N_{vortices} = \alpha S\sqrt{\phi_0}$$

where S is the area of the region, α is a numerical constant.

2.4 The decay of vortices by radiation and the suppression of a large convection cell

The convection cell is destroyed when

- the internal vortical motion radiates energy through linear drift waves and so it loses the structure of flow
- the Reynolds stress of the background turbulence is sufficient to produce sheared plasma rotation. This destroys the radial correlation of the flow in the convective flow and reinstate the small scale turbulence

For an estimation of the radiation of energy from a cuasi-soliton the field is Fourier transformed along the coordinate of the propagation, y. The equation of the vortex in the region around the core, at small amplitude, is, for Fourier modes on y,

$$\frac{\partial^2\phi_{k_y}}{\partial x^2} + \left[-\left(\frac{1}{T(x)} - \frac{v_{dia}(x)}{u}\right) - k_y^2\right]\phi_{k_y} = 0$$

The two zeros of the "potential" (coefficient of ϕ_{k_y}) suggests to use the WKB method and

$$\phi_{k_y} = A_{k_y}\ Q_{k_y}^{-1/4}(x,u)\ \exp\left[i\int_{x_T}^{x} dx'\ Q_{k_y}^{1/2}(x',u)\right]$$

where

$$\begin{aligned} Q_{k_y}(x,u) &> 0 \\ Q_{k_y}(x,u) &= \frac{v_{dia}(x)}{u} - \frac{1}{T(x)} - k_y^2 \end{aligned}$$

The density of energy of the vortex is

$$\mathcal{E} = \frac{1}{2}\left[(\boldsymbol{\nabla}_\perp\phi)^2 + \frac{\phi^2}{T(x)}\right]$$

Note
This comes from the origibal form of the equation

$$\begin{aligned} & \left(\frac{1}{T(x)} - \boldsymbol{\nabla}_\perp^2\right)\frac{\partial\phi}{\partial t} \\ & + \left(v_{dia} + \frac{\partial v_{dia}}{\partial x}\ x\right)\phi \\ & -\kappa_T\phi\frac{\partial\phi}{\partial y} \\ & - \left[(-\boldsymbol{\nabla}_\perp\phi\times\widehat{\mathbf{n}})\cdot\boldsymbol{\nabla}_\perp\right]\boldsymbol{\nabla}_\perp^2\phi \\ = & \ 0 \end{aligned}$$

multiplied by ϕ and written as

$$\mathcal{E} = \frac{1}{2}\left[\frac{\phi^2}{T(x)} + (\boldsymbol{\nabla}_\perp\phi)^2\right]$$

and

$$\frac{\partial\mathcal{E}}{\partial t} + \boldsymbol{\nabla}_\perp\cdot\mathbf{S} = 0$$

where $\mathbf{S}$ is the flux of energy.
End.

Integrating over volume we obtain the static energy

$$\mathcal{E}_{vol} = \int \mathcal{E}\ d^3x = \text{const}\times\frac{u^4 k_0}{\left(\frac{\partial v_{dia,0}}{\partial x}\right)^2}\left(\frac{4}{3} + k_0^2\right)$$

When we calculate the time variation of $\mathcal{E}_{vol}$, one has to replace in the integrand (in the density of energy) the full expression of the vortex core, with linear displacement along y. The result (**Horton Su Morrison**) is

$$\begin{aligned} \frac{d\mathcal{E}_{vol}}{dt} &\sim -b^{1/3} \\ b &= \frac{4k_0^3}{3\left|\alpha\right|} \end{aligned}$$

According to this formula the time of decay of a vortex is

$$\tau = \frac{2.8\pi^2}{u\left|\alpha\right|}\left(\frac{4}{3}+k_0^2\right)\ \exp\left[\frac{4k_0^3}{3\left|\alpha\right|}\right]$$

since k_0 is usually very small [$k_0^2 = 1 - \frac{v_{dia,0}}{u}$, the vortex moves with a speed that is higher but close to the diamagnetic velocity, *i.e.* the "effective Larmor radius" is very large], the time of decay is proportional with u^{-1}, the fastest vortices decay the first: those vortices that propagate with speed not very much above the diamagnetic one will survive for longer time, for them the effective Larmor radius is extremly large, they are in Euler-type dynamics.

3 Drift-Alfven vortices at the ion Larmor radius

We now turn to investigate the possibility of drift-Alfven vortices that have spatial transversal dimension of the order of ρ_i. There is no "electric velocity" v_E. The equilibrium magnetic field is $\mathbf{B}_0 = B_0\widehat{\mathbf{e}}_z$. The fields $\mathbf{E}$ and $\mathbf{B}$ are *perturbations.*

$$\begin{aligned} E_z = \mathbf{E}\cdot\widehat{\mathbf{e}}_z &= -\frac{\partial A}{\partial t} - \frac{\partial \phi}{\partial z} \\ \mathbf{E}_\perp &= -\boldsymbol{\nabla}_\perp \phi \\ \mathbf{B}_\perp &= \boldsymbol{\nabla} A \times \widehat{\mathbf{e}}_z \end{aligned}$$

This aspect is very important. The poloidal flow in tokamak is usually the result of a radial electric field E_r whose origin is to be found outside the setting that we examine. It can be loss of fast NBI ions, etc. It is an input parameter. However in the present problem it is not included, E_r is only due to the perturbation.

3.0.1 For the *electrons*

The electron velocity in *low frequency approximation*

$$\mathbf{v}_e = \mathbf{v}_E + \mathbf{v}_{e,dia} + v_{z,e}\left(\widehat{\mathbf{e}}_z + \frac{\mathbf{B}_\perp}{B_0}\right)$$

$$\mathbf{v}_{e,dia} = -\frac{T_e}{n_0 m_i}\frac{1}{\Omega_i}\widehat{\mathbf{e}}_z \times \boldsymbol{\nabla}_\perp n_0$$

The z component of the current

$$\begin{aligned} j_z &= -\frac{1}{\mu_0}\nabla_\perp^2 A \\ &= -en_0 v_{z,e} \end{aligned}$$

which gives

$$v_{z,e} = \frac{1}{en_0\mu_0}\nabla_\perp^2 A$$

To use now the electron continuity equation one has to note that

$$\boldsymbol{\nabla}\cdot(n_e \mathbf{v}_{e,dia}) = 0$$

and

$$\boldsymbol{\nabla}\cdot\mathbf{v}_E = 0$$

then

$$\frac{\partial n_e}{\partial t} + \boldsymbol{\nabla}_\perp (n_e \mathbf{v}_e) + n_0\left(\frac{\partial}{\partial z} + \frac{\mathbf{B}_\perp}{B_0}\cdot\boldsymbol{\nabla}\right) v_{z,e} = 0$$

Note the derivative along the "perturbed" magnetic field line, *i.e.* taking into account the deviation of the line from $\mathbf{B}_0$ by the field $\mathbf{B}_\perp$.

Introduce the notation for the deviation of the density from equilibrium

$$n_e = n_0 + \widetilde{n}_e$$

then the electron equation of continuity is

$$\begin{aligned} & \frac{1}{n_0}\left(\frac{\partial}{\partial t} + \mathbf{V}_E\cdot\boldsymbol{\nabla}\right)\widetilde{n}_e \\ & + v_{e,dia}\frac{\partial}{\partial y}\left(\frac{e\phi}{T_e}\right) \\ & + v_A^2\ \rho_s^2\quad\left(\frac{\partial}{\partial z} + \frac{\mathbf{B}_\perp}{B_0}\cdot\boldsymbol{\nabla}\right)\nabla_\perp^2\left(\frac{eA}{T_e}\right) \\ = & \ 0 \end{aligned}$$

The first term is convection of the perturbed density by the *electric* velocity.

The second term is the poloidal convection by the diamagnetic velocity $v_{e,dia}\frac{\partial}{\partial y}$ of the Boltzmann distribution of perturbed density.

The last term is the ballistic modification $\sim \partial/\partial z$ of the parallel velocity of the electrons $v_{ez} \sim j_{ez} \sim \boldsymbol{\nabla}_\perp^2 A_z$ along the magnetic field line (deviated from $\mathbf{B}_0$ by $\mathbf{B}_\perp$)

We conclude that the electron density perturbation $\widetilde{n}_e$ is not the Boltzmann distribution $e\phi/T_e$.

$$\frac{\widetilde{n}_e}{n_0} = \frac{e\phi}{T_e} - \frac{u}{\alpha}\left(1 - \frac{v_{dia,e}}{u}\right)\frac{eA}{T_e}$$

The equation for momentum of electrons

It is written along the full magnetic field

$$\mathbf{B}_0 + \mathbf{B}_\perp$$

as

$$0 = -\left(\frac{\partial}{\partial z} + \frac{\mathbf{B}_\perp}{B_0} \cdot \boldsymbol{\nabla}\right) \widetilde{p}_e - e n_0 E_\parallel$$

(no electron inertia term, no collisional friction) and the notation is

$$E_\parallel = E_z + \frac{\mathbf{B}_\perp}{B_0} \cdot \mathbf{E}_\perp$$

The equation of parallel momentum can be written

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + v_{e,dia}\frac{\partial}{\partial y}\right)\left(\frac{eA}{T_e}\right) \\ & + \left(\frac{\partial}{\partial z} + \frac{\mathbf{B}_\perp}{B_0} \cdot \boldsymbol{\nabla}\right)\left(\frac{e\phi}{T_e} - \frac{\widetilde{n}_e}{n_0}\right) \\ = & \ 0 \end{aligned}$$

This was for electrons. The Lagrangian change of A in the poloidal flow is balanced by the parallel variation of the defect (the non-Boltzmannian part of the electron density perturbation).

3.0.2 Now, for the ions

The ion momentum equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \boldsymbol{\nabla}\right) \mathbf{v}_i = -\frac{1}{m_i n_i} \boldsymbol{\nabla}_\perp p_i + \frac{e}{m_i}\mathbf{E} + \frac{e}{m_i}\mathbf{v}_i \times \mathbf{B}$$

There is a small parameter due to the separation of time scales

$$\frac{\left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \boldsymbol{\nabla}\right)}{\Omega_i} \ll 1$$

which results

$$\begin{aligned} \mathbf{v}_i \ = \ & \mathbf{v}_E + \mathbf{v}_{i,dia} \\ & + \frac{1}{\Omega_i}\widehat{\mathbf{e}}_z \times \left\{ \left[\frac{\partial}{\partial t} + (\mathbf{v}_E + \mathbf{v}_{i,dia}) \cdot \boldsymbol{\nabla}\right] (\mathbf{v}_E + \mathbf{v}_{i,dia}) \right\} \end{aligned}$$

the square paranthesis is the scalar operator of convective derivation (convection by electric and dia velocities) and this term is the iteration of the zero order velocity $(\mathbf{v}_E + \mathbf{v}_{i,dia})$, which leads to polarization velocity.

The definition

$$\mathbf{v}_{i,dia} = \frac{1}{n_i m_i}\frac{1}{\Omega_i}\widehat{\mathbf{e}}_z \times \boldsymbol{\nabla}_\perp p_i$$

Consider the explicit form of the "polarization" term

$$\begin{aligned} & \frac{1}{\Omega_i}\widehat{\mathbf{e}}_z \times \left\{ \left[\frac{\partial}{\partial t} + (\mathbf{v}_E + \mathbf{v}_{i,dia}) \cdot \boldsymbol{\nabla} \right] (\mathbf{v}_E + \mathbf{v}_{i,dia}) \right\} \\ = \ & \mathbf{v}_{i,E}^{pol} + \mathbf{v}_{i,dia}^{pol} \end{aligned}$$

with the new notations

$$\mathbf{v}_{i,E}^{pol} = \frac{1}{\Omega_i}\widehat{\mathbf{e}}_z \times \left[\left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \boldsymbol{\nabla} \right) \mathbf{v}_E \right]$$

The scalar operator of convective time derivative is written in detail

$$\begin{aligned} & \frac{\partial}{\partial t} + \mathbf{v}_E \cdot \boldsymbol{\nabla} \\ = \ & \frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial \left(e\phi / T_i \right)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \left(e\phi / T_i \right)}{\partial y} \frac{\partial}{\partial x} \right) \end{aligned}$$

Then

$$\mathbf{v}_{i,E}^{pol} = -\rho_i^2 \left\{ \frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial \left(e\phi / T_i \right)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \left(e\phi / T_i \right)}{\partial y} \frac{\partial}{\partial x} \right) \right\} \boldsymbol{\nabla}_{\perp} \left(\frac{e\phi}{T_i} \right)$$

The other polarization velocity

$$\begin{aligned} \mathbf{v}_{i,dia}^{pol} = \ & \frac{1}{\Omega_i}\widehat{\mathbf{e}}_z \times \left\{ \left[\frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial \left(e\phi / T_i \right)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \left(e\phi / T_i \right)}{\partial y} \frac{\partial}{\partial x} \right) \right] \mathbf{v}_{i,dia} \right. \\ & \left. + \mathbf{v}_{i,dia} \cdot \boldsymbol{\nabla} \left(\mathbf{v}_E + \mathbf{v}_{i,dia} \right) \right\} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_{i,dia}^{pol} = \ & -\frac{\rho_i^2}{n_0} \left[\frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial \left(e\phi / T_i \right)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \left(e\phi / T_i \right)}{\partial y} \frac{\partial}{\partial x} \right) \right] \boldsymbol{\nabla}_{\perp} n_i \\ & -\frac{\rho_i^3}{n_0} v_{th,i} \left[\frac{\partial n_i}{\partial x} \frac{\partial \ \boldsymbol{\nabla}_{\perp} \left(\frac{e\phi}{T_i} \right)}{\partial y} - \frac{\partial n_i}{\partial y} \frac{\partial \ \boldsymbol{\nabla}_{\perp} \left(\frac{e\phi}{T_i} \right)}{\partial x} \right] \end{aligned}$$

We now have the explicit form of the ion velocity

$$\mathbf{v}_i = \mathbf{v}_E + \mathbf{v}_{i,dia} + \mathbf{v}_{i,E}^{pol} + \mathbf{v}_{i,dia}^{pol}$$

and we can write the equation of continuity for the ion density

$$\frac{\partial n_i}{\partial t} + \boldsymbol{\nabla}_{\perp} \cdot (n_i \mathbf{v}_i) = 0$$

and it results

$$\begin{aligned} & \left[\frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial \left(e\phi / T_i \right)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \left(e\phi / T_i \right)}{\partial y} \frac{\partial}{\partial x} \right) \right] n_i \\ & + n_i \boldsymbol{\nabla} \cdot \left(\mathbf{v}_{i,E}^{pol} + \mathbf{v}_{i,dia}^{pol} \right) \\ = \ & 0 \end{aligned}$$

Again it is separated a perturbation

$$n_i = n_0 + \widetilde{n}_i$$

and it results

$$\begin{aligned}
& \left[\frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial (e\phi/T_i)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial (e\phi/T_i)}{\partial y} \frac{\partial}{\partial x} \right) \right] \left(1 - \rho_i^2 \boldsymbol{\nabla}_\perp^2 \right) \widetilde{n}_i \\
& -u_{*i}\ n_0\ \frac{\partial}{\partial y} \left(\frac{e\phi}{T_i} \right) \\
& -n_0 \rho_i^2 \left[\frac{\partial}{\partial t} + \rho_i v_{th,i} \left(\frac{\partial (e\phi/T_i)}{\partial x} \frac{\partial}{\partial y} - \frac{\partial (e\phi/T_i)}{\partial y} \frac{\partial}{\partial x} \right) \right] \nabla_\perp^2 \left(\frac{e\phi}{T_i} \right) \\
= \quad & \rho_i^3 v_{th,i} \left[\frac{\partial (\nabla_\perp n_i)}{\partial x} \frac{\partial\ \boldsymbol{\nabla}_\perp \left(\frac{e\phi}{T_i}\right)}{\partial y} - \frac{\partial (\nabla_\perp n_i)}{\partial y} \frac{\partial\ \boldsymbol{\nabla}_\perp \left(\frac{e\phi}{T_i}\right)}{\partial x} \right]
\end{aligned}$$

The first line is

$$\begin{aligned}
& \left\{ \frac{\partial}{\partial t} + \left[\left(-\boldsymbol{\nabla}_\perp \phi \times \widehat{\mathbf{e}}_z \right) \cdot \boldsymbol{\nabla} \right] \right\} \widetilde{n}_i \\
- & \left\{ \frac{\partial}{\partial t} + \left[\left(-\boldsymbol{\nabla}_\perp \phi \times \widehat{\mathbf{e}}_z \right) \cdot \boldsymbol{\nabla} \right] \right\} \rho_i^2 \boldsymbol{\nabla}_\perp^2 \widetilde{n}_i
\end{aligned}$$

We note that, if the density perturbation $\widetilde{n}_i = -\frac{e\phi}{T_i}$ the first convective term vanishes and the second term becomes equal to the third line described below.

The second line is

$$v_{i,dia} \frac{\partial}{\partial y} \quad \rightarrow \quad \text{diamagnetic poloidal convection of the Boltzmann part of } \widetilde{n}_i$$

The third line is the electric convection of the polarization velocity

$$n_0 \left\{ \frac{\partial}{\partial t} + \left[\left(-\boldsymbol{\nabla}_\perp \phi \times \widehat{\mathbf{e}}_z \right) \cdot \boldsymbol{\nabla}_\perp \right] \right\} \rho_i^2 \nabla_\perp^2 \left(\frac{e\phi}{T_i} \right)$$

The RHS is

$$\rho_i^3 v_{th,i} \left\{ \frac{\partial}{\partial t} + \left[-\boldsymbol{\nabla}_\perp \left(\boldsymbol{\nabla}_\perp \widetilde{n}_i \right) \times \widehat{\mathbf{e}}_z \right] \cdot \boldsymbol{\nabla}_\perp \right\} \boldsymbol{\nabla}_\perp \left(\frac{e\phi}{T_i} \right)$$

It is a kind of convection derivative operator, but the "electric" velocity that usually is obtained from $E \times B$ now is obtained from an *equivalent* of the electric potential,

$$\frac{e\phi}{T_i} \sim \nabla_\perp \widetilde{n}_i$$

NOTE

The difference between $\widetilde{n}_i$ and $\frac{e\phi}{T_i}$ is the contribution from the Alfven perturbation, which involves A_z.

This is the reason for which the ion density response is NOT Boltzmannian.

But when the Larmor radius of the ions is very large (*i.e.* $\rho_{eff} \rightarrow \infty$) the ion response is Boltzmannian, since the A_z part is almost suppressed.

END

NOTE

Here we would obtain the *response* $\widetilde{n}_i$ of the ion density to a potential ϕ in diamagnetic velocity u_{*i}.

This response we expect to be dependent on the difference

$$u_{*,i} - V_E$$

between the diamagnetic and the electric velocities.

We expect that, when this difference becomes very small, the ion density response $\widetilde{n}_i$ becomes adiabatic, which means that the perturbation is Boltzmannian.

This would allow to use this in the study of the vortical filaments of **Petviashvili Pogutse** which are electron vortices and their existence depends on the Boltzmann response of the ions, which only occurs if the (effective) Larmor radius is extremely large.

END

The neutrality equation

$$\widetilde{n}_e = \widetilde{n}_i$$

consider the vortex

$$(x, y) \rightarrow (x, \eta = y - ut + \alpha z)$$

$$\begin{aligned} u \quad &\equiv \quad \text{speed of translation of the vortex along the } y \text{ axis} \\ &\quad\quad \text{(poloidal)} \end{aligned}$$

The equations

$$\begin{aligned} &\left\{ \frac{\partial}{\partial \eta} - \frac{\rho_s c_s}{u} \left[-\boldsymbol{\nabla} \left(\frac{e\phi}{T_e} \right) \times \widehat{\mathbf{e}}_z \right] \cdot \boldsymbol{\nabla} \right\} \left(\frac{\widetilde{n}_e}{n_0} - \frac{v_{e,dia}}{u} \frac{e\phi}{T_e} \right) \\ = \quad &\frac{\alpha v_A^2 \rho_s^2}{u} \left\{ \frac{\partial}{\partial \eta} - \frac{\rho_s c_s}{\alpha} \left[-\nabla \left(\frac{eA}{T_e} \right) \times \widehat{\mathbf{e}}_z \right] \cdot \boldsymbol{\nabla} \right\} \nabla_\perp^2 \left(\frac{eA}{T_e} \right) \end{aligned}$$

and

$$\begin{aligned} &\left\{ \frac{\partial}{\partial \eta} - \frac{\rho_s c_s}{\alpha} \left[-\nabla \left(\frac{eA}{T_e} \right) \times \widehat{\mathbf{e}}_z \right] \cdot \boldsymbol{\nabla} \right\} \left[-\frac{u}{\alpha} \left(1 - \frac{v_{e,dia}}{u} \right) + \frac{e\phi}{T_e} - \frac{\widetilde{n}_e}{n_0} \right] \\ = \quad &0 \end{aligned}$$

The equation for $\Phi_i = \frac{e\phi}{T_i}$ is

$$\frac{1}{\kappa^2}\nabla_\perp^4\left(\frac{e\phi}{T_i}\right) - \nabla_\perp^2\left(\frac{e\phi}{T_i}\right) - C\left(\frac{e\phi}{T_i} - uB_0 x\right) = 0$$

$$\kappa^2 = -\frac{\left(u^2 - uv_{dia,i} - \alpha^2 v_A^2\right)\left(1 - \frac{v_{dia,e}}{u}\right)}{\alpha^2 v_A^2 \rho_i^2 \left(1 + \frac{T_e}{T_i}\right)}$$

the solution $e\phi/T_i$ scales as

$$\exp\left(-\kappa r\right)$$

$$k_\perp^2 \to -\kappa^2$$

The transversal localization is stronger for $v_{dia,i} \to u$.

The propagation velocity of the drift-Alfven vortices

$$\begin{array}{lll} u & > & 0 \text{ ion drift modes} \\ u & < & 0 \text{ electron drift modes} \end{array}$$

The transversal dimension of the vortex

$$\frac{L}{\rho_i}$$

is higher and higher as

$$\frac{u}{v_{dia,i}}$$

increases.

When the diamagnetic velocity is close to u the scale is small, the vortex is localized. But there is another parameter

$$\mu_* \equiv \frac{\alpha v_A}{v_{dia,i}}$$

4 Electron solitary vortices smaller than ρ_i

This is a problem first examined by Petviashvili Pogutse.

The fluctuations of electrons have scales of the order of the

$$\delta \equiv \text{skin length}$$

and produce transport. The reason is that at such scales the electrons are not frozen in the magnetic field.

The paper shows that are possible vortices of electrons with transversal dimension smaller than ρ_i.

this may be highly relevant to us, if the "Larmor radius" of ions that is involved in this phenomenon is an effective one.

The vortices are insensitive to magnetic shear. They move with a velocity that is smaller than the "drift" velocity (diamagnetic). **Note** that this does not involve yet the two velocities: rotation and diamagnetic. It is just the velocity of displacement of vortices.

Where intervenes the fact that their dimension is $< \rho_i$? It is in the distribution of ions at these scales: it is Boltzmann.

If ϕ is the potential of the vortex of electrons, the ions have the density

$$n = n_0 \left(1 + \kappa x - \frac{T_e}{T_i}\phi\right)$$

the parameters are assumed constant

$$\begin{aligned} \kappa &\sim \frac{1}{L_n} = \text{const} \\ \tau &\equiv \frac{T_e}{T_i} = \text{const} \end{aligned}$$

The density of ions is linear in $\frac{1}{L_n}$ (this simply means a constant gradient, as in pedestal) and is Boltzmann in ϕ.

We now must prove that the fact that the ion density is Boltzmann over the scale of the transverse section of a vortex (not yet found) is derived from the fact that the ion effective Larmor radius is very large, or

$$v_E - v_{i,dia} \approx 0$$

In other words it is sufficient that the effective Larmor radius ρ_{eff} to be large enough for any perturbation inside this effective Larmor radius to produce an ion Boltzmann response (see **NOTE** above).

This allows us to consider the role of such vortices as primary manifestations of the Edge Localized Modes. They convert the layer of plasma poloidal rotation into a set of filament-vortices that actually destroy the transport barrier and allow a pulse of high loss.

5 Conclusions

The objective of this work is the examination of the "Running scale separation" in a mixed state of turbulence and coherent structures. The large scale structures are sometimes observed in experiments and are known to provide higher transport rates since they connect distant regions in plasmas. We focus on the assumption that the origin of large scale structures is the formation, attraction and merging of nonlinear smaller structures, vortices. Although the vortices can appear in strong turbulence as spontaneous local organization of the nonlinear field (flow) we consider here the situation where the formation of vortices is triggered by an extrinsic pulse, therefore the vortices are transient. Large spatial scale structures are transiently generated and determine intermittently enhanced transport.

Using classical analytical formulations, we have written the explicit form of vortices and of their dissipative loss of structure via drift wave radiation. We have estimated the number of vortices that can be created from an extrinsic (perturbing) pulse arriving at the plasma edge from an event located in the core.

In relation with the main subject (generation of vortices and their evolution via coalescence to coherent convective cells) we have formulated a related problem.

The conditions near the plasma edge in the regime of high confinement (H-mode) are favorable to the periodic edge localize modes. A theoretical approach indicates the "peeling balloonning" instability as the explanation for these events. However in H mode the plasma has strong rotation and this almost suppresses the peeling-ballooning instability.

We suggest that in the stage of high efficient transport barrier, with a high pedestal and with strong plasma rotation there is natural condition for the formation of vortices. They break the continuity of the pedestal barrier and place the plasma in favorable condition for the peeling ballooning instability which further will lead to ELMs. The vortices however only exist if the transversal section is smaller than the ion Larmor radius.

We have shown that the ion Larmor radius that is involved in the constraint is an "effective" one,

$$\frac{1}{\rho_{eff}^2} = \frac{1}{\rho_s^2}\left(1 - \frac{v_{dia}}{v_\theta}\right)$$

This is indeed very large since in the pedestal the plasma rotation is higher than the diamagnetic velocity but the latter increases continuously due to the rise of gradient in the transport barrier. Then the constraint found by **Petviashvili Pogutse** is verified and vortices will grow in the rotation layer.

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